1.4 Problem Set 1

Success in understanding Structural Mechanics requires that students should be competent in certain mathematical skills. This problem set gives a very small number of typical mathematical problems that students should be able to solve quickly and confidently. Problem Set 2 covers the other part of the assumed knowledge – statics and equilibrium.

- Solving algebraic equations,
- Differentiation and integration (including double integrals),
- Drawing graphs of polynomials (especially) and other mathematical functions,
- Trigonometry.

1) Given $N_1 = (A_1/A_2) \bullet N_2$ and $N_1 + N_2 = P$, solve for N_1 and N_2 in terms of the other variables.
2) $\int \sin ax\, dx$.
3) $\iint x^2 y^2\, dxdy$.
4) If $d^4y/dx^4 = x$, what is y?
5) If $v = \dfrac{w}{24\,E\,I}\left(z^4 - 2\,L\,z^3 + L^3 z\right)$, what is d^2v/dz^2.
6) Draw the following 2 graphs in the region $-2 < x < 2$, showing zeroes, and any relevant maxima or minima.
   i. $y = 4 + 2x - 2x^2$
   ii. $y = x^3 + x^2 - 2x$
7) An equilateral triangle has sides 200 mm. What is its height?

These are all very simple questions, but it is possible that students may be a little rusty on some basic mathematical skills, and hence this should be seen as an activity to jolt the mathematical brain back into action. Students with genuine difficulties should see the lecturer as soon as possible.

2.7 Problem Set 2

2.7.1 Part A – Fundamentals

1) For each of the following question, draw the free body diagram and determine the reaction forces and moments.

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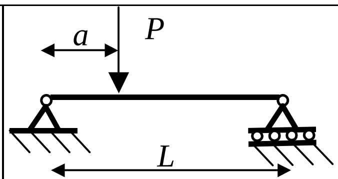
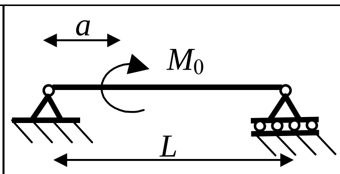
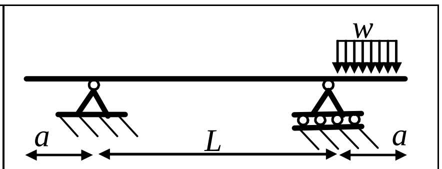
<tr><td>(a)</td><td>
</td><td>(b)</td><td>
</td><td>(c)</td><td>
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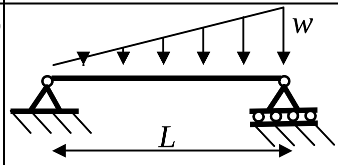
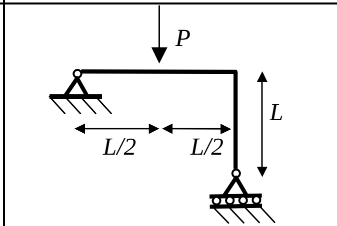
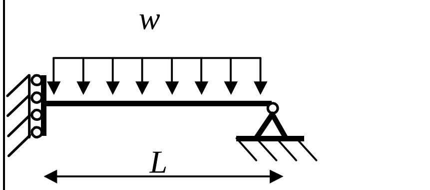
<tr><td>(d)</td><td>
</td><td>(e)</td><td>
</td><td>(f)</td><td>
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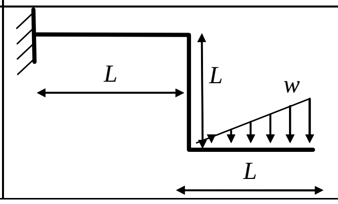
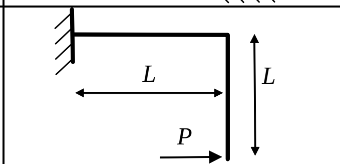
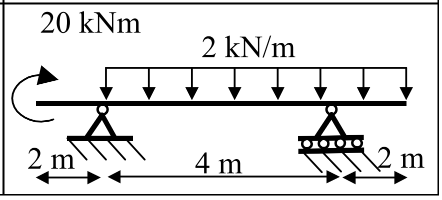
<tr><td>(g)</td><td>
</td><td>(h)</td><td>
</td><td>(i)</td><td>
</td></tr>
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2.7.2 Part B – Applied Questions

2) For each of the following question, draw the free body diagram and determine the reaction forces and moments. *Ability to answer questions of this nature quickly and accurately is assumed knowledge.*

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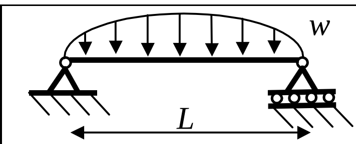
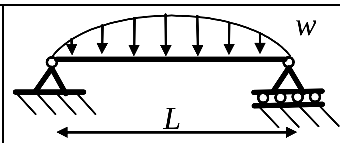
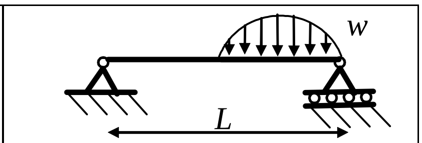
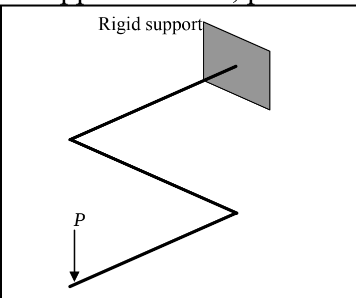
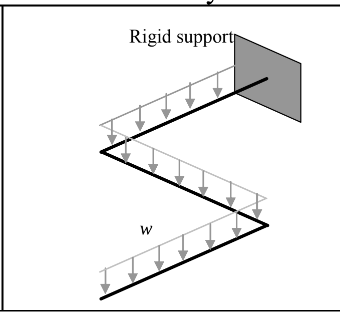
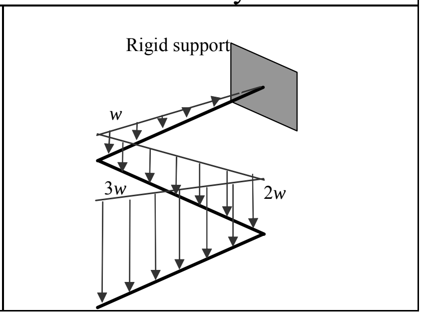
<tr><td>(a)</td><td>

Parabolic distributed load, maximum intensity w at the midpoint</td><td>(b)</td><td>

Sinusoidally distributed load, maximum intensity w at the midpoint</td><td>(c)</td><td>

Sinusoidally distributed load over half the beam, maximum intensity w at the midpoint of the load distribution.</td></tr>
<tr><td colspan="6">In the following questions the structure is made up of components, each of length L. The structure lies in a horizontal plane and each segment is perpendicular to the adjacent segment. The support is fixed, preventing deflection in every direction, and rotation about any axis.</td></tr>
<tr><td>(d)</td><td>
</td><td>(e)</td><td>
</td><td>(f)</td><td>
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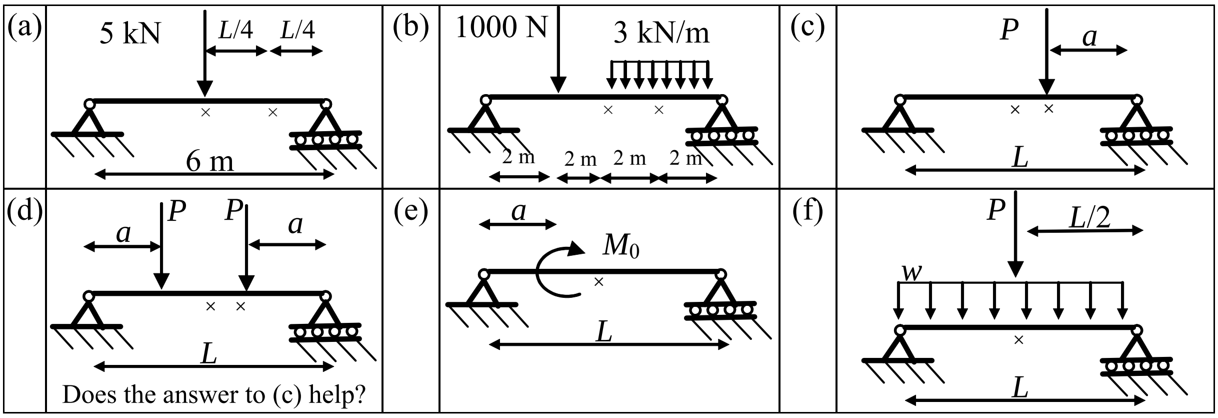
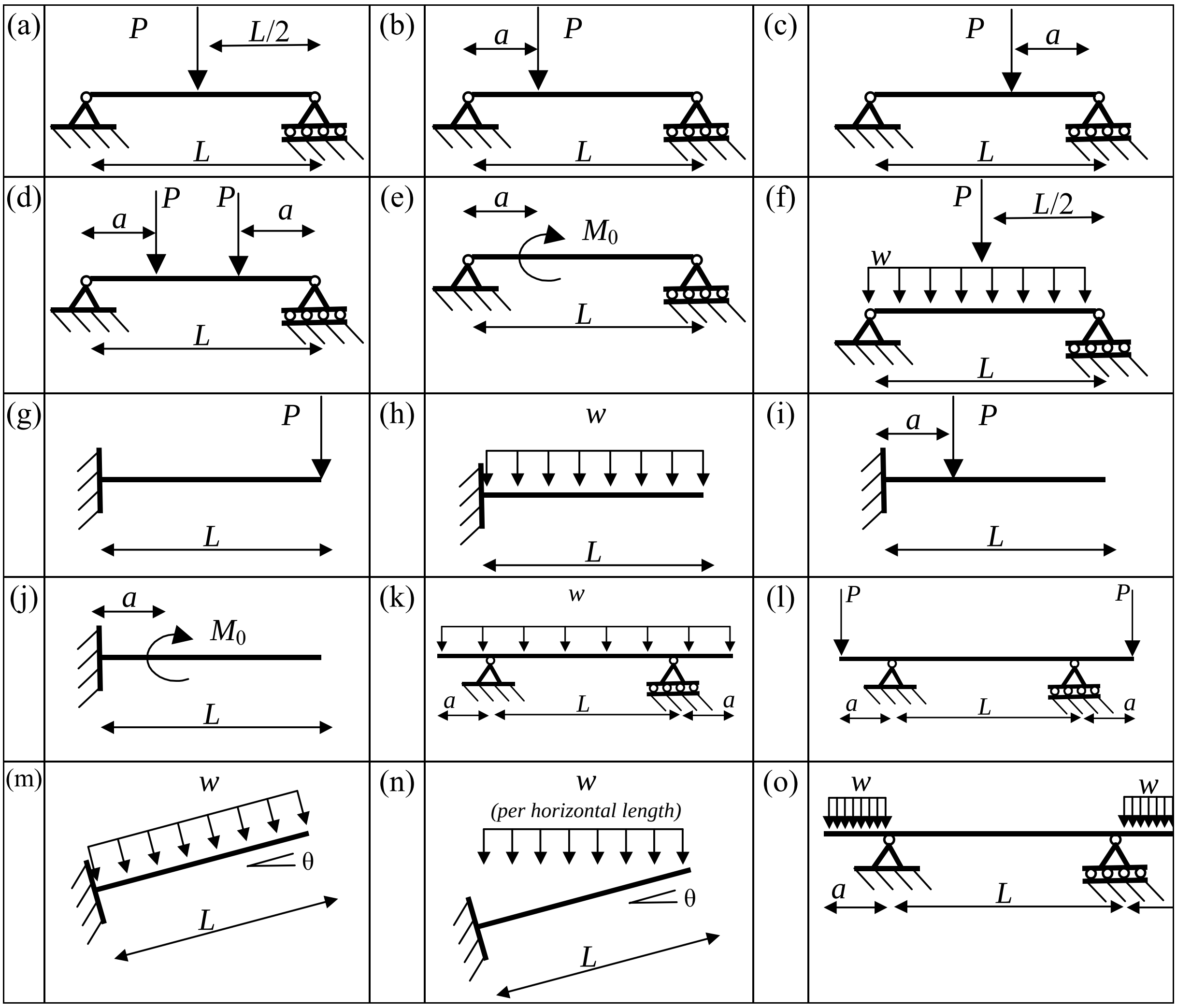
3.10 Problem Set 3

3.10.1 Part A – Fundamentals

1) Determine the magnitude of the bending moment & shear force at the locations shown.



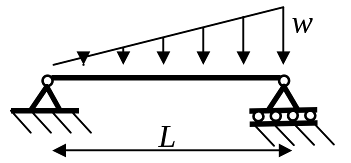
2) Draw the BMD, SFD, and AFD for the following cases. Indicate the value and location of the maximum bending moment/shear force/axial force in each case.



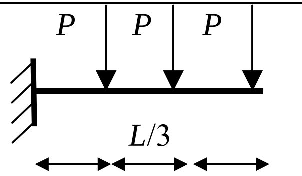
Part B – Applied Questions

3) Draw the BMD, SFD, and AFD for the following cases. Indicate the value and location of the maximum bending moment/shear force/axial force in each case.

(a)



(b)



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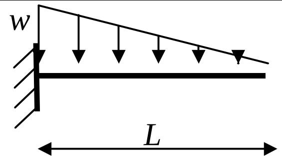

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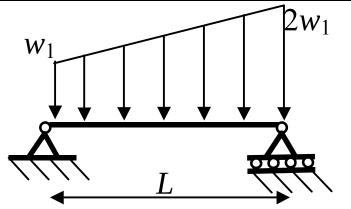

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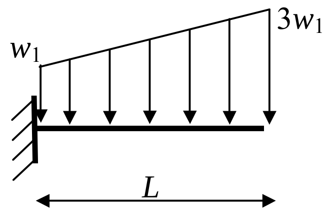

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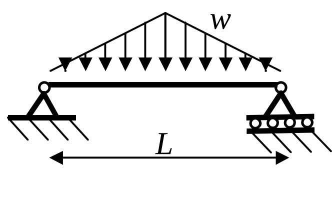

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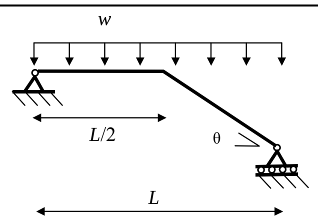

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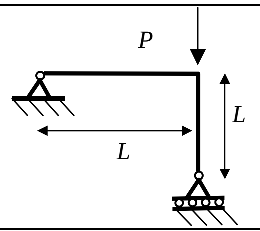

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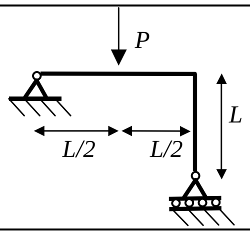

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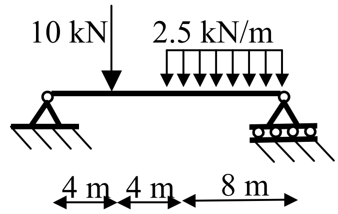

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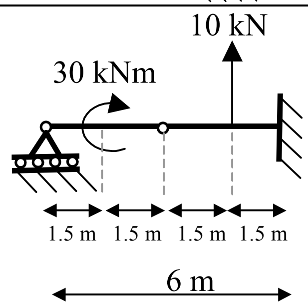

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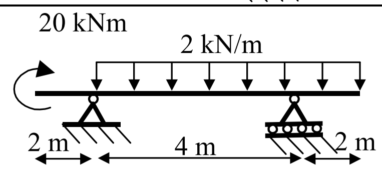

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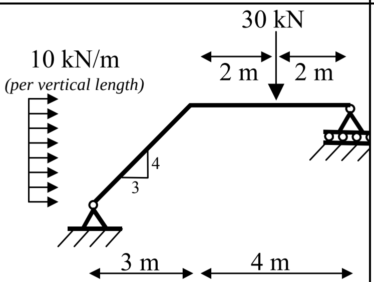

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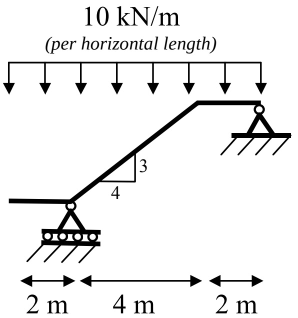

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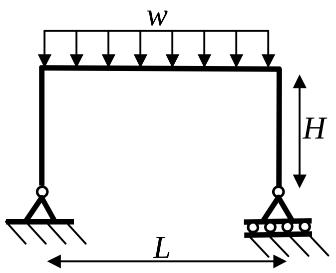
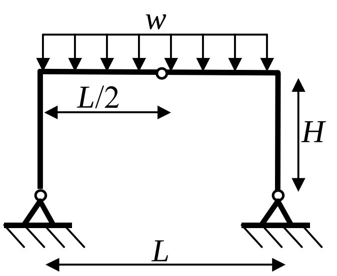
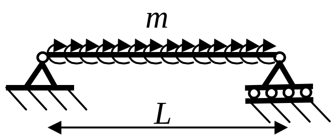


(p)

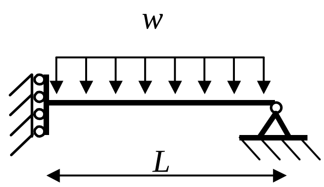


(q)

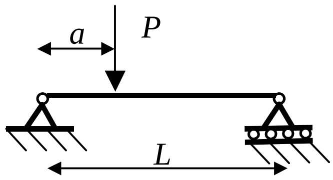


m is a uniformly distributed moment (eg 10 kNm/m)

(r)



4) An American builder has a large supply of round timber rod, 2 inches in diameter. The timber has a density of 650 kg/m^3. The builder wants use a long piece of timber as a beam but must ensure that the maximum moment due to the self-weight of the beam does not exceed 75 Nm.
   a) What is the maximum length simply supported beam the builder could produce?
   b) What is the maximum length cantilever beam the builder could produce?

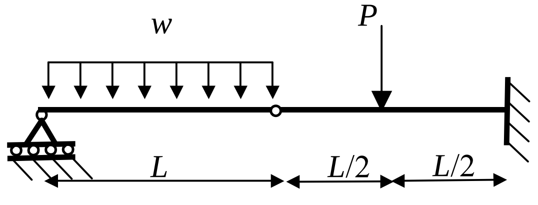
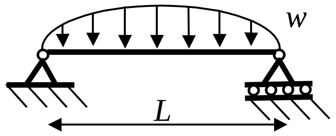


5) Consider the simply supported beam shown.
a) What is the value of a which produces the maximum bending moment in the beam? What is the value of this moment?
b) What is the value of a which produces the minimum maximum shear force in the beam? (Think what this is asking). Denote this V_{min}.
c) For an arbitrary value of a, what magnitude of point load (W) is required to produce the same value of maximum bending moment that was calculated in part (a) (W should be a function of P)? Plot the ratio W/P (y axis) against a for $0 < a < L$.
d) For the values of a and W considered in (c) above, what is the maximum shear force in the beam? Denote this value V_{max}. Plot the ratio V_{max}/V_{min} (y axis) against a for $0 < a < L$.

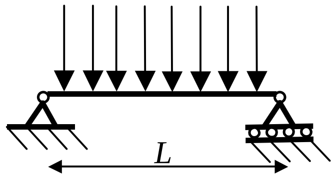
6) A sign-pole is 10 m tall and supports a sign that is 3 m wide and 2 m high. The top of the sign is at the same level as the top of the pole. The sign is connected to the post in 2 places, one is 200 mm from the top of the sign, and the other is 200 mm from the bottom of the sign. It can be assumed that each connection transfers the same amount of load from the sign to the post. The support (base) of the pole is completely fixed, preventing deflection in every direction, and rotation about any axis. Wind imparts a uniform pressure of 1 kPa perpendicular to sign. Draw the free body diagram clearly showing all the reactions at the base of the pole; calculate all the reactions; and draw the twisting moment diagram and bending moment diagram for the pole.

3.10.2 Part C – Advanced Questions

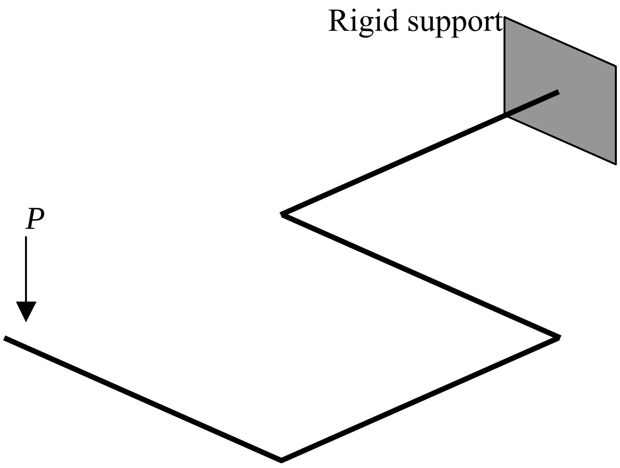
7) Draw the BMD, SFD, and AFD for the following cases. Indicate the value and location of the maximum bending moment/shear force/axial force in each case.

(a)		(b)	 Parabolic distributed load, maximum intensity w at the midpoint

8) The simply supported beam has n equal point loads, equally spaced along the length of the member. The total load is equal to P. Derive a general expression for the maximum bending moment in the beam in terms of n, P, and L. Plot the variation of M_{max} against n. Is there a limiting value as $n \to \infty$? If so what is it and is there anything significant about it?



9) The square hook-shaped structure is made up of four components, each of length L. The structure lies in a horizontal plane and each segment is perpendicular to the adjacent segment. There is a vertical load P at the end of the structure. The support is completely fixed, preventing deflection in every direction, and rotation about any axis. Draw the bending moment, twisting moment, and shear force diagrams for the structure.



4.13 Problem Set 4

1) Research (by the internet or the library) to tabulate the basic material properties (E, f_y) for as many common materials as possible. This would include metals such as steel and aluminium, timbers, plastics, concrete, glass etc.

2) Consider an elastic – perfectly plastic material. The modulus of elasticity is 120 000 MPa, and yield stress is 240 MPa. Draw the stress strain curve, and determine the stresses when the strain has the following values
   a. 0.0005
   b. 0.001
   c. 0.002
   d. 0.003
   e. 0.005

Show these points on the stress strain curve.

5.6 Problem Set 6

5.6.1 Part A – Basic Questions

1) An engineer is assessing the suitability of three different materials for possible use as a tension member in a lightweight truss. The truss member is 3 m long and must resist a tensile load of 200 kN. It can be assumed that the bars are elastic.

- Option 1: Steel bar, area = 500 mm^2, E = 210000 MPa.
- Option 2: Copper bar, area = 750 mm^2, E = 120000 MPa.
- Option 3: Aluminium bar, area = 1500 mm^2, E = 70000 MPa.

For each case, determine the axial extension of the bar. Which bar experiences the least extension?

2) A timber column with constant circular cross section, radius 200 mm, is subjected to an axial compression. The column is 10 m tall. The lower half of the column is made from timber with an elastic modulus 10000 MPa, while the upper half of the column is made from timber with an elastic modulus of 12000 MPa. Section A is 2 m from the top of the column, and section B is 1.5 m from the bottom of the column.
   a) For an axial compression of 120 kN, what is the axial shortening?
   b) What applied axial compression, P, will produce a total axial shortening of 2.5 mm?
   c) Determine the stress and strain at the centre of the column at cross-sections A & B for the load P calculated in part (b)?

3) Consider a solid aluminium bar, 25 mm diameter, with properties $E_{aluminium}$ = 70000 MPa and $\alpha_{aluminium} = 24 \times 10^{-6}$ (ºC)$^{-1}$ which is 1000 mm long. Determine the stress, strain and force in the bar when the temperature of the bar is increased from 20 ºC to 50 ºC for the following two sets of end conditions:

a) Bar is restrained at one end only

b) Bar is restrained at both ends

5.6.2 Part B – Intermediate Questions

4) A compound bar consists of a solid aluminium bar, 25 mm diameter, encased by a steel tube, 25 mm inner diameter and 30 mm outer diameter. The steel has properties E_{steel} = 200000 MPa and $\alpha_{steel} = 12 \times 10^{-6}$ (ºC)$^{-1}$, while the aluminium has properties $E_{aluminium}$ = 70000 MPa and $\alpha_{aluminium} = 24 \times 10^{-6}$ (ºC)$^{-1}$. The bar is 1000 mm long. Determine the stress, strain and force in the steel and aluminium components of the compound bar when the temperature of the bar is increased from 20 ºC to 50 ºC for the following two sets of end conditions:

a) Bar is restrained at one end only

b) Bar is restrained at both ends

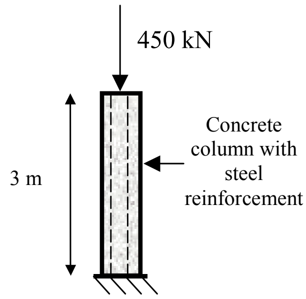
5) A square concrete column, 200 mm × 200 mm is symmetrically reinforced by four steel reinforcing bars of diameter 12 mm. The column is 3 m tall and is subjected to an axial compression of 450 kN. It can be assumed that E_{steel} = 200000 MPa, and $E_{concrete}$ = 33333 MPa, and that $f_{y,\ steel}$ = 500 MPa and $f_{y,\ concrete}$ = 25 MPa. The materials behave elastically until yield.

a) Calculate the stress in the steel and concrete due to the 450 kN compression.

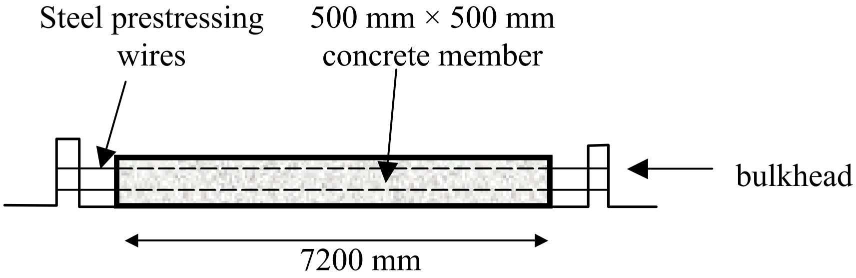
b) Calculate the total force resisted by the steel and the total force resisted by the concrete.

c) What is the axial shortening of the column due to the 450 kN load?

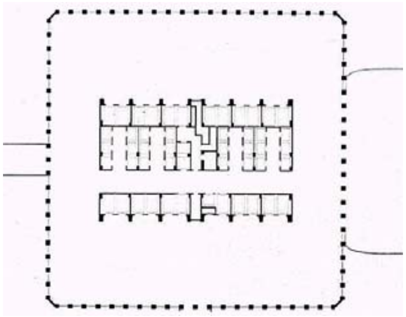
d) The axial load is increased. Which material will yield first – steel or concrete? At what value of axial load will this occur?



6) A 500 mm by 500 mm square concrete member is prestressed concentrically in the horizontal position by the release of the anchorage on tensioned steel wires. A force of 320 kN is introduced into the wires which have a total cross sectional area of 3000 mm^2 and are anchored to bulkheads as shown before the concrete is poured. Compute the loss of prestress force in the steel due to the elastic shortening of the concrete when the steel wires are cut off from the bulkheads at the ends of the beam after curing of the concrete, neglecting the shrinkage of concrete due to drying and creep. It may be assumed that E_{steel} = 200000 MPa, and $E_{concrete}$ = 20000 MPa. The member is 7200 mm long.



5.6.3 Part C – Advanced Questions



7) An engineer has been asked to assess the performance of the columns in the World Trade Center. A typical floor plan is shown on the left. Several assumptions are made:

- The external columns and the core carry half the dead load each.
- The dead load can be considered as uniformly distributed up the height of each tower.
- The height of each tower can be assumed to be 1360 feet, and each tower has a mass of 1000000 (metric) tonnes.
- There are 236 external columns, each column is a square hollow section, 450 mm × 450 mm (external) and thickness 12.5 mm. (This is incorrect, the thickness of the columns tapers from 12.5 mm at the bottom to 7.5 mm at the top – but this assumption makes it easier).

   Calculate the axial shortening (in mm) of a typical column under self-weight of the structure.

8) Archie the architect wants to build the world's tallest flagpole. Archie plans to use an 88.9 × 4.8 C350 CHS as the flagpole (steel pole, circular hollow section, outer diameter 88.9 mm, thickness 4.8 mm, yield stress $f_y = 350$ MPa). Assuming that steel has a density of 7850 kg/m^3, what is the tallest flagpole Archie could design while ensuring that the maximum compressive stress in the pole due to the self weight did not exceed the yield stress? What is the axial shortening associated with this height of flagpole? Archie is not happy with this height and seeks the second opinion of Ernie the engineer. Archie asks Ernie if choosing a bigger pole would help (eg thicker or larger diameter). What advice should Ernie give Archie on this question? What possible solutions could Ernie suggest to Archie as to how to make a taller flagpole?

   (The only relevant loading is self-weight, and buckling should not be considered – it has not been covered yet in this bok)

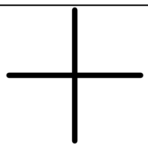
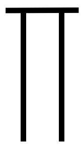
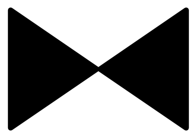
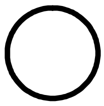
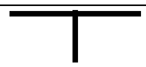
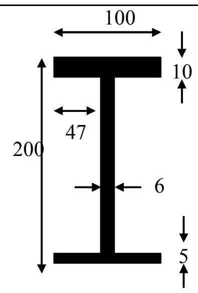
9) Reconsider question 3. One material will yield first, and then behave plastically. However the second material will continue to behave elastically until it reaches its yield stress. At what load will the second material reach its yield stress? Draw a load vs axial shortening graph for the column.

10) Recalculate the answer to Question 7, assuming the thickness of the SHS varies linearly from 12.5 mm at the bottom, to 7.5 mm at the top.

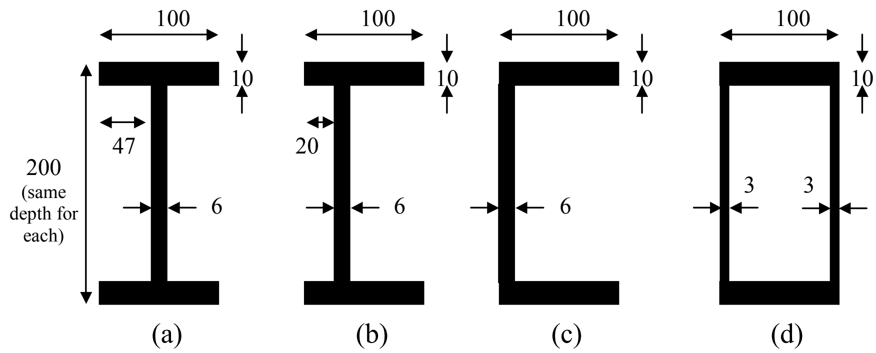
6.12 Problem Set 6

6.12.1 Part A – Basic Questions

1) Find the location of the centroid, and determine the second moments of area about horizontal and vertical centroidal axes, for the following cross-sections. Are these axes the principal axes? For each of the sections, calculate (separately) the bending moments about the horizontal and vertical centroidal axes that would produce a maximum bending stress of 150 MPa in the cross section?

 (a) Cruciform section Thickness 6 mm, Total height = total width = 90 mm	 (b) Flange (horizontal) is 50 × 6 mm Each web (vertical) is 100 × 5 mm Gap between webs is 50 mm	 (c) Each solid equilateral triangle has sides of length 1 m.
 (d) Circular hollow section Outer diameter 100 mm, thickness 4 mm	 (e) Flange (horizontal) is 100 × 6 mm Web (vertical) is 55 × 5 mm	 (f) Asymmetric I-section (mm)

2) Calculate the 2nd moment of area about the horizontal centroidal axis for the following figures. Is there a trick to these questions? Draw the location of the horizontal centroidal axis.

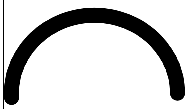
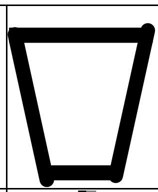
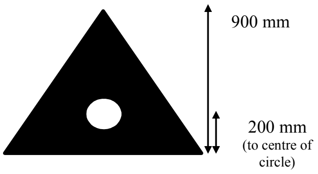
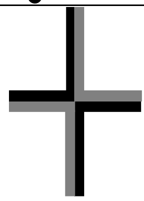


All dimensions are in mm
Each shape is symmetric about a horizontal axis of symmetry

3) Visit the OneSteel website and obtain the dimensions and properties of the 610UB113 section. Calculate the value of I_x from the dimensions given and compare it to the value given in the catalogue. Assuming the steel has a yield stress of 280 MPa, calculate the bending moment about the x-axis that will initiate yielding in the section. Draw the distribution of stress and strain across the section associated with this value of moment.

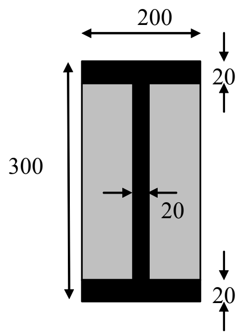
6.12.2Part B – Intermediate Questions

4) Find the location of the centroid, and determine the second moments of area about horizontal and vertical centroidal axes, for the following cross-sections:

	(a) Semi-circular arc, 15 mm diameter, 3 mm thick.		(b) Large precast concrete box section: 800 mm wide at top, 500 mm wide at bottom, 1100 mm high & 75 mm constant thickness
	(c) Equilateral triangle, with circular cut-out (100 mm diameter)		(d) Cruciform section made from 4 × 150 × 100 × 12 UA sections. The longer legs are vertical. Information on the properties of this section can be found at in the appendix of the lecture notes.

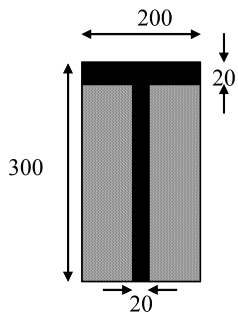
5) A solid rectangular timber member has cross-sectional dimensions of 300 mm × 200 mm. The member is experiencing an axial compression of 300 kN, a bending moment of 11 kNm about the major principal axis, and a bending moment of 0.1 kNm about the minor principal axis. Identify the points on the cross-section that experience the maximum and minimum values of stress and calculate these values of stress.

6) An aluminium I beam (shown in black) has been strengthened by securely attaching it to two timber sections (shown in grey). Given that E_{timber} = 10 GPa and E_{alumnium} = 70 GPa, and that a positive bending moment of 30 kNm about the horizontal centroidal axis is applied, calculate the maximum tensile stress experienced by the timber, and the maximum compressive stress experienced by the steel. Indicate where these stresses occur. Also draw (with values) the distribution of stress and strain across the section.



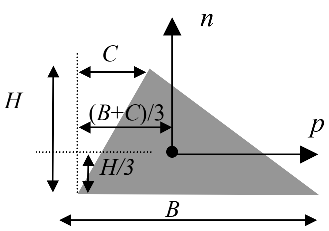
All dimensions are in mm

7) An aluminium T beam has been strengthened by securely attaching it to two timber sections as shown. Given that E_{timber} = 12.5 GPa and E_{alumnium} = 70 GPa, and that a positive bending moment of 30 kNm about the horizontal centroidal axis is applied, calculate the maximum tensile stress experienced by the timber, and the maximum compressive stress experienced by the steel. Indicate where these stresses occur.



All dimensions are in mm

8) Given I_{horz} of the following triangle, use the parallel axis theorem to derive I_{vert}.



$$I_p = \frac{BH^3}{36}$$

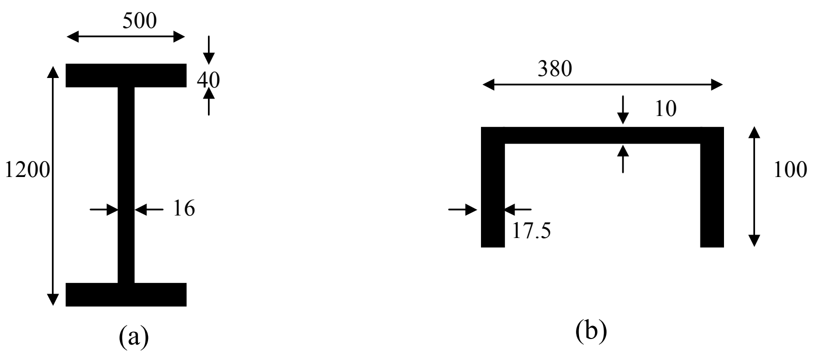
(Note, *horz* & *vert* are not necessarily the principal axes)

6.12.3 Part C – Advanced Questions

9) Consider the reinforced concrete beam in Example 6.8.1. It was found that the concrete cracked when there was an applied moment of 0.173 kNm. Determine the location of the neutral axis of the cracked section, and determine the stress and strain distribution if a moment of 0.5 kNm is applied about the horizontal axis.

10) From first principles (ie integration), derive the value of the second moment of area of a circle.

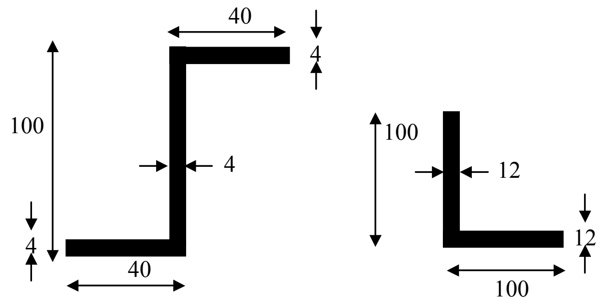
11) For the following cross-sections, determine the elastic and plastic section moduli for bending about a horizontal axis. Assuming the material has elastic-plastic section properties, with yield stress f_y = 350 MPa, determine the bending moment that causes first yield, and the full plastic moment. Also give the elastic and plastic section moduli for bending about the horizontal axis. Draw a diagram showing the location of the elastic and plastic neutral axes in each case.



All dimensions are in mm
Note different scales on each section

<u>Approximate</u> answers: (a) M_y = 8900 kNm, M_p = 9900 kNm; (b) M_y = 31 kNm, M_p = 56 kNm;

12) For the following sections, draw a diagram showing the location and orientation of the principal axes and determine the second moments of area about the *principal* axes.

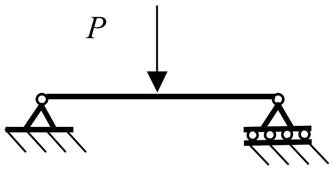


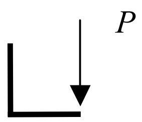
(a) (b)

All dimensions are in mm
Note different scales on each section

Approximate answers: (a) $\theta = 17^\circ$, $I_x = 1.0 \times 10^6$ mm^4, $I_y = 60 \times 10^3$ mm^4;
(b) $\theta = 45^\circ$, $I_x = 3.3 \times 10^6$ mm^4, $I_y = 0.83 \times 10^6$ mm^4.

13) A 100 × 100 × 12 equal angle is used as a beam. The beam is loaded at midspan, and the load acts at the end of the horizontal leg of the angle. Without performing any calculations, draw the cross section at midspan as the beam deforms under load. The scale is unimportant, but the diagram should clearly show how the cross section will deflect. The cross section will not distort (ie the shape of the cross-section itself does not change, but the whole cross-section will move





Cross section at midspan

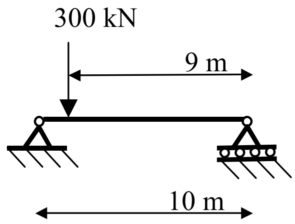
Assuming that the length of the beam is 3 m, and the load $P = 7$ kN, determine the vertical deflection of the "heel" of the angle section

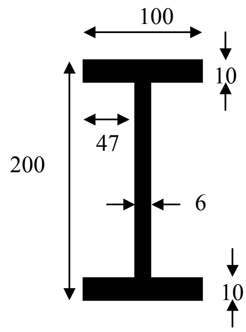
Approximate Answer: Vertical Deflection = 2.3 mm.

7.6 Problem Set 7

7.6.1 Part B – Intermediate Questions

1) An I-section is used as a simply supported as shown below.
   a) Draw the BMD and SFD of the beam.
   b) Based on the maximum shear force in the beam, determine (and draw) the shear stress distribution in the I-section.
   c) Approximate the shear stress by dividing the shear force by the area of the web, and compare this value with those calculated above.





All dimensions are in mm

Typical beam cross-section

The remaining questions are on bending.

2) Consider a square cross section $d \times d$ and assuming a yield stress f_y determine the yield moment for bending about the horizontal axis. Repeat the process for a square with twice the area, for a rectangle with dimensions $2d \times d$, and a rectangle with dimensions $d \times 2d$ (height × width). Rank these 4 sections in order of efficiency (yield moment relative to area).

3) Consider a circular hollow section with outer diameter 300 mm, and thickness 2 mm. Determine A, I_x and Z_x of this section. Consider other cross-sections which have the same area – a solid circle; a solid square; a solid rectangle (height = 3 × width); a square hollow section with thickness 2 mm; an I section with web and flange thickness of 2 mm and flange width equal to web depth; and an I section with web thickness 1 mm, flange thickness 2 mm, and flange width half of the web depth. Determine A, I_x and Z_x of these other sections and produce a table that ranks them in order of I_x and Z_x.

7.6.2 Part C – Advanced Questions

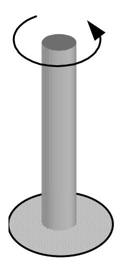
4) Consider a solid circular cross-section with diameter D_0. Calculate the area and second moment of area and call these A and I_{x0}. The diameter of the circle is increased while maintaining a constant area by making the section hollow. There is now a circular hollow section with diameter D_1, thickness t_1 and area A. Calculate the second moment of area and denote it I_{x1}. Plot the ratio I_{x1}/I_{x0} against D_1/D_0 for $1 < D_1/D_0 < 10$.

8.7 Problem Set 8

8.7.1Part A – Basic Questions

1) Answer the following questions:
- Calculate the torsion constant, J, for a solid circular section with a diameter of 0.25 m.
- Calculate the torsion constant, J, for a hollow circular section with a diameter of 0.25 m, and thickness 50 mm.
- Calculate the thickness of a hollow circular section, diameter 0.25 m, that has a J value that is 50 % of the J value for a solid circular section of diameter 0.25 mm.

2) A solid circular cylinder, 500 mm in height and with a diameter 50 mm, is subjected to a twisting moment of 20 kNm. The cylinder is made from a material with shear modulus G = 50000 MPa.

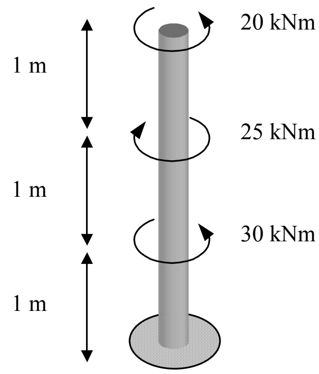


- Determine the torsion constant J.
- Draw the twisting moment diagram.
- Calculate the rate of twist and the maximum shear stress.
- What is the rotation of the top of the cylinder with respect to the fixed base?

What is the rotation if the material is changed to another material with G = 100000 MPa?
Are any of the previous results affected by the different material (J, TMD, τ, rate of twist)?

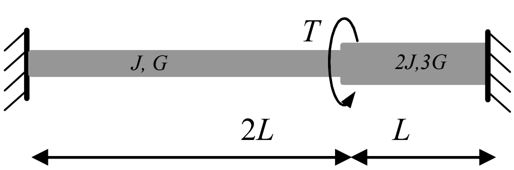
8.7.2 Part B – Intermediate Questions

3) A steel pole is made from 3 circular hollow sections each 1 m in length. Each section of the pole has an outer diameter of 300 mm, but the thickness varies. The bottom third has thickness 6 mm, the central third has thickness 4 mm, and the top third has thickness 2 mm. The shear modulus of steel is G = 80000 MPa.



- Draw the twisting moment diagram.
- Calculate the maximum shear stress.
- Calculate the rate of twist in each section of the pole.
- Draw a diagram that shows the rotation with respect to the fixed base at any height.

4) Two solid circular rods, one of length 2L, with properties G & J, the other of length L and properties 3G and 2J are joined together and rigidly connected to supports at the free ends. A torque T is applies at the junction of the two tubes. Draw the TMD (*hint: Example 5.4.1)*.



8.7.3 Part C – Advanced Question

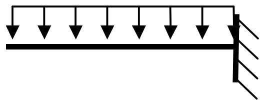
5) Consider a solid circular cross-section with diameter D_0. Calculate the area and torsion constant and call these A and J_0. The diameter of the circle is increased while maintaining a constant area by making the section hollow. There is now a circular hollow section with diameter D_1, thickness t_1 and area A. Calculate the torsion constant and denote it J_1. Plot the ratio J_1/J_0 against D_1/D_0 for $1 < D_1/D_0 < 10$.

9.5 Problem Set 9

9.5.1 Part A – Basic Questions

1) An American builder has a large supply of timber with cross sectional dimensions of 50 mm (high) × 25 mm (wide). The timber has a density of 1100 kg/m^3. The builder must ensure that the maximum deflection due to the self-weight of the beam does not exceed $L/500$ where L is the length of the beam. Assuming that the timber has elastic modulus $E = 10$ GPa, what is the maximum length simply supported beam that the builder could produce?

2) Standard case – find an equation for the deflected shape, and the value of the maximum deflection for a cantilever of length L, with a uniformly distributed load of magnitude w.



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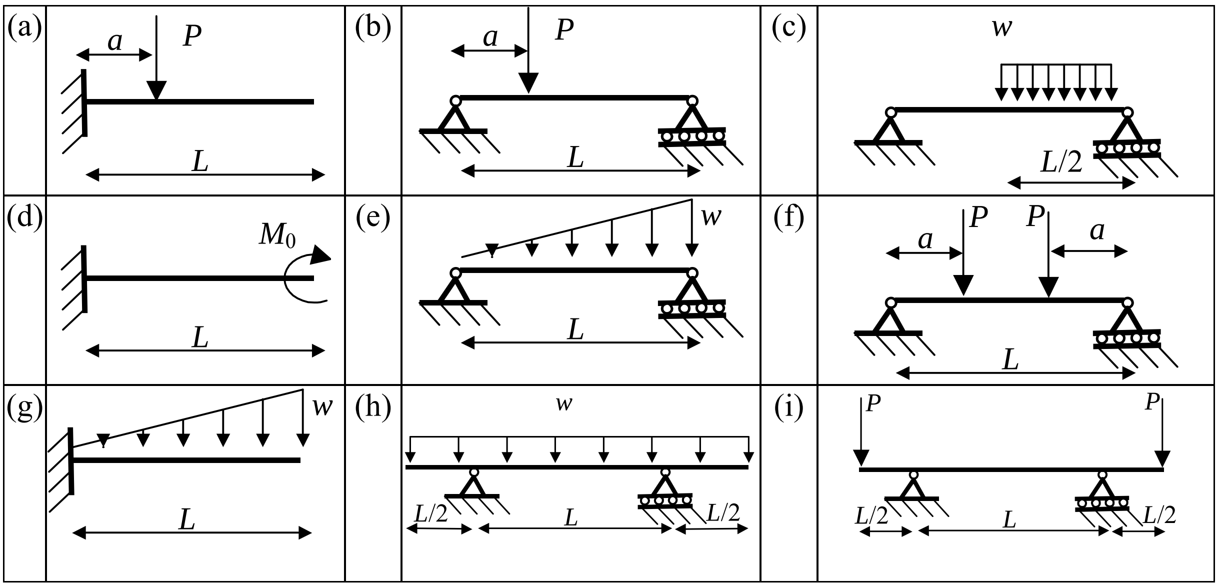
9.5.2 Part B – Intermediate Questions

3) For the following cases

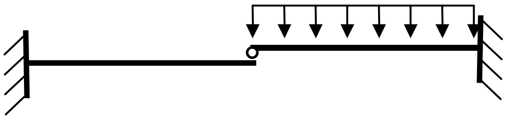
- Write an algebraic expression which defines either the loading, the bending moment distribution or the shear force (whichever is applicable) – Macauley brackets may need to be used;
- Write down the appropriate boundary conditions
- Integrate the appropriate differential equations, apply the boundary conditions, and hence determine an expression for the deflected shape.
- Determine the maximum deflection (positive and negative if appropriate).

For each case the beam has constant properties, E and I.



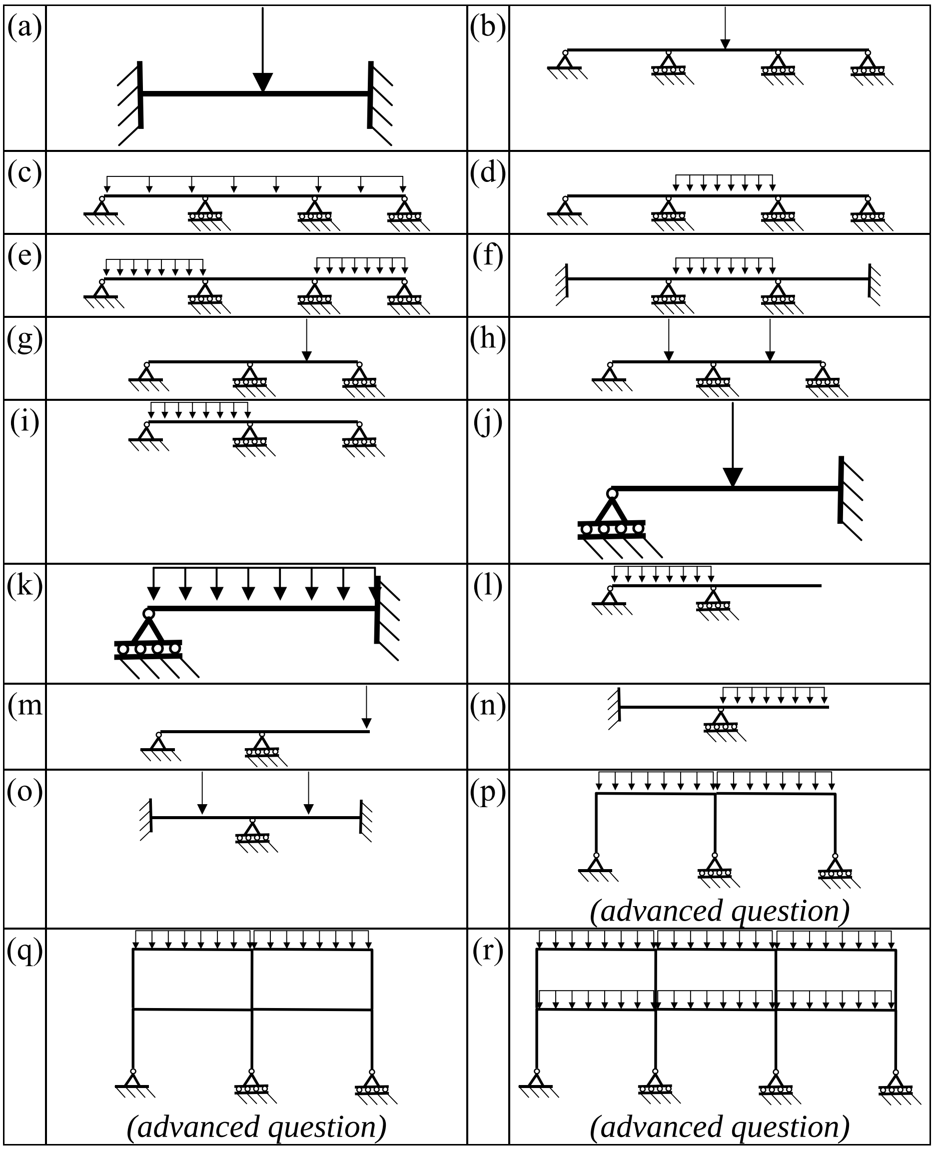
These questions, while not excessively hard, can take some time. It is suggested that students attempt two or three of these questions, such as (c), (d), and (e).

4) Consider two identical cantilevers, each of length L, with elastic modulus E, and second moment of area I. The tip of one cantilever just rests on the other by means of a roller. The "upper" cantilever is loaded by a UDL of magnitude w.



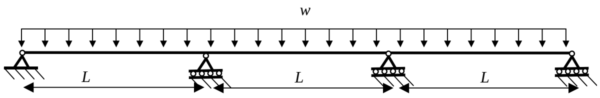
   - Draw the FBD of each cantilever.
   - What *compatibility* condition(s) exist to help analyse the two beams?
   - Use compatibility to determine the reaction force between the tips of cantilever.
   - Determine the deflection at the cantilever tips, and determine the BMD/SFD (with values) for each cantilever.

5) Without performing any calculations, sketch the deflected shape and then the BMDs and SFDs in the following cases. (It can be assumed that all loads have the same magnitude, all spans are the same length, and all point loads are applied either at midspan or endspan).



9.5.3 Part C – Advanced Questions

6) Using the philosophy of Example 9.2.7, determine the maximum deflection in each span, and also draw the BMD and SFD (values required).



10.3 Problem Set 10

10.3.1 Part A – Basic Questions

1) A large pipe has a diameter of 2 m, and a design pressure of 600 kPa. What minimum pipe thickness will ensure that the hoop stress is no more than 50 MPa?

2) Consider a closed cylindrical pressure vessel, diameter 500 mm, and thickness 5 mm. The internal pressure is 30 MPa. Determine the longitudinal and hoop stresses and draw a diagram that shows the stresses a typical element experiences.

10.3.2 Part B – Intermediate Question

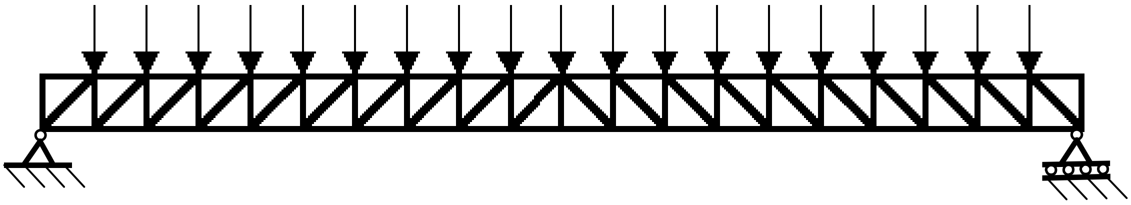
3) Consider a spherical pressure vessel with diameter D and thickness t. Draw a diagram that shows the distribution of stress in a typical element of the vessel. Using a similar procedure to that in the Lecture Notes, derive an expression for the stresses in the vessel in terms of the pressure P, D and t.

11.7 Problem Set 11

11.7.1 Part B – Intermediate Question

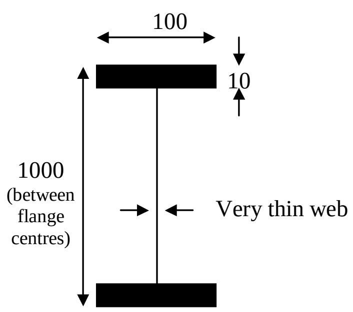
1) Consider the truss shown below. The truss has a total length of 20 m. There are 20 panels and each panel is 1 m deep. A point load of 1 kN is applied at each of the panel points on the top chord as shown.

    - Identify the members of the truss that will experience the maximum tension and compression.
    - Use an appropriate method to determine the values of the maximum tension and compression.



The truss is then replaced by a beam, 20 m long, with an equivalent UDL acting. The web of the beam can be considered to be so thin that the web makes no contribution to the area or the stiffness of the section.

- What is the UDL that is equivalent to the 1 kN forces acting at each panel point 1 m apart?
- Determine the maximum bending moment in the beam.
- Determine I_x for the beam.
- Determine the stress at the centre of each flange beam due to the bending moment.
- Calculate the average force in each flange, by multiplying the tensile or compressive stress at the midpoint of the flange by the area of the flange.
- Compare these values with the maximum tension and compression forces in the truss and discuss.

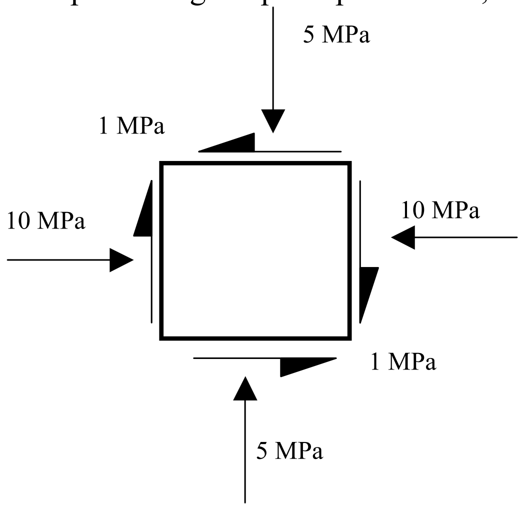


All dimensions are in mm

12.4 Problem Set 12

12.4.1 Part A – Basic Question

1) The stresses on an element in a structure have been calculated and are shown. Calculate the principal stresses, the maximum shear stress, the orientation of the planes experiencing the principal stresses, and draw the Mohr's Circle.



12.4.2 Part B – Intermediate Question

2) Consider a closed cylindrical pressure vessel, diameter 500 mm, and thickness 5 mm. The cylinder is 5 m long. The internal pressure is 2.5 MPa. The cylinder also experiences a constant twisting moment of 50 kNm along its entire length. Determine the longitudinal, hoop and shear stresses and draw a diagram that shows the stresses a typical element experiences. Hence calculate the principal stresses, the maximum shear stress, the orientation of the planes experiencing the principal stresses, and draw the Mohr's Circle. The material has a yield stress of 300 MPa. Determine whether the material has yielded according to both Tresca's and von Mises' yield criteria.

13.4 Problem Set 13

13.4.1 Part A – Basic Question

1) The "London Eye" (see picture on right) is a large observation wheel of 135 m diameter, on the banks of the River Thames in London. The large rotating wheel is supported by an A-frame constructed from steel circular hollow sections that have a 3000 mm outer diameter and are 40 mm thick. It can be assumed that E_{steel} = 200000 MPa.

a) Calculate the second moment of area (I) of the circular hollow sections used in the A-frame. Give the answer in both metric units and non-metric units (it may assumed that 1 inch = 25.4 mm).

b) Assuming that each member of the A-frame can be considered as a column with an effective length of 70 m, calculate the Euler buckling load of one of the columns.

13.4.2 Part B – Intermediate Question

2) The areas of the universal beam 410UB59.7 and the universal column 200UC59.3 are approximately the same, and the section properties can be found from the OneSteel website.

- List the area (A_g), the second moments of area about both axes (I_x & I_y), and the elastic moduli (Z_x & Z_y) for these two sections (include units).
- Calculate the Euler buckling loads for buckling about both the *x*- & *y*- axes assuming an effective length (L_e) of 5 m for both buckling about both axes.
- Calculate the bending moment to cause first yield (assuming f_y = 300 MPa) for both sections for bending about both the *x* and *y* axes.
- Based on your answers, comment on the relative performance in compression of universal beam sections compared to universal column sections. Also comment on the relative performance in bending of universal beam sections compared to universal column sections.