The University of Sydney

School of Civil Engineering

CIVL2201 Structural Mechanics

Semester 1, 2009

Time Allowed: 3 hours + 20 minutes reading time

Instructions to Candidates

(a) This paper contains 3 parts - A, B, & C; and is marked out of 90.

- **Part A has 3 questions (Q 1 – Q 3, 30 marks).**
- **Part B has 3 questions (Q 4 – Q 6, 60 marks).**
- **Part C has 2 questions (Q 7 - Q 8, 20 marks - which are bonus marks).**

(b) All questions may be attempted.

(c) Suitable working, diagrams and explanations are required for each question.

(d) Marks may be deducted for work that is not satisfactorily set out.

(e) Units are important.

(f) Programmable and non-programmable calculators may be used.

(g) Read the questions carefully before answering.

(h) **Annotated copies of the "Structural Mechanics" Text Book may be taken into the exam, but no other written material is permitted.** Other sheets of paper may not be inserted into the set of lecture notes.

(i) Each question has an "explain" type component. It is expected that students should answer these parts with a short written explanation (one or two paragraphs) plus a diagram. Performing calculations is not necessarily expected, but using equations might also be useful.

Part A – Fundamental Questions

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1) 10 Marks

Consider an overhanging beam (such as problem 2)o) on page 67 of the text book). The total length of the beam is 10 m, the span between supports is 8 m, and the length of each overhang is 1 m. There is a uniformly distributed load (UDL) of 1 N/mm, acting vertically downwards between the supports only (ie the UDL acts over a length of 8 m), and there are two point loads, each of magnitude 4 kN acting vertically downwards at each of the two free ends. The effects of self-weight are negligible compared to the effects of the applied loading. The beam is made from steel which has an elastic modulus of 200000 MPa.

(a) **Draw the free body diagram** of the beam clearly showing the support reactions. **Calculate** the values of the support reactions.

(b) **Draw the bending moment diagram (BMD) and shear force diagram (SFD). Clearly indicate the values** of bending moment and shear force at the key points along the beam (the supports, midpoint and free ends).

(c) It is generally assumed that steel is *linear elastic*. With the aid of diagrams, and by specifically referring to the results of Lab Session 2, **briefly explain what *linear elastic* behaviour means with respect to a steel beam under load**.

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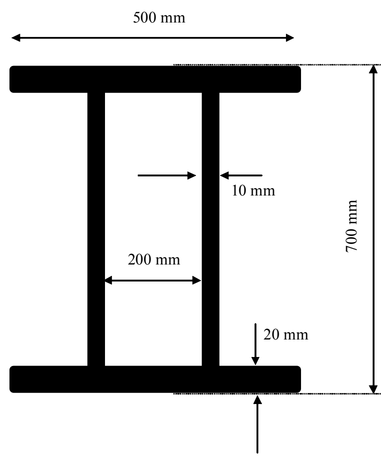
2) 10 Marks



Consider the following modified box section, the dimensions of which are shown in the diagram. The section is doubly-symmetric. The cross section is made from aluminium which has a Young's Modulus of 70000 MPa, and a yield stress of 280 MPa.

(a) **Calculate the second moment of area** (I_x) of the section about the horizontal centroidal axis.

(b) **Calculate the bending moment** about the horizontal axis that will cause first yield of the section.

(c) **Draw the stress and strain distributions** (values required) in the cross-section due to the bending moment calculated in (c) above.

(d) The designer tries to improve the efficiency of the beam by maintaining the same depth of section and volume of material, and making the flanges thinner and appropriately wider. **Comment on whether this modification will affect the strength of the section in *x*-axis bending (part b) and explain the answer**.

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3) 10 marks

In Example 9.2.3 of the lecture notes, it is shown that the deflection of a simply supported beam with a UDL is given by $v = \frac{w}{24EI}\left(z^4 - 2Lz^3 + L^3 z\right)$.

Consider a 360UB56.7 steel beam, with a span of 9 m, under its self-weight only. The relevant page of the section properties handbook is given on the back page on this examination sheet – students are free to use the values given in that table without the need to recalculate them. The density of steel is 7850 kg/m^3 and the elastic modulus is E = 200000 MPa. Linear elastic material properties should be assumed. The beam is oriented so that its web is in the vertical plane. Gravity may be assumed as 10 m/s^2.

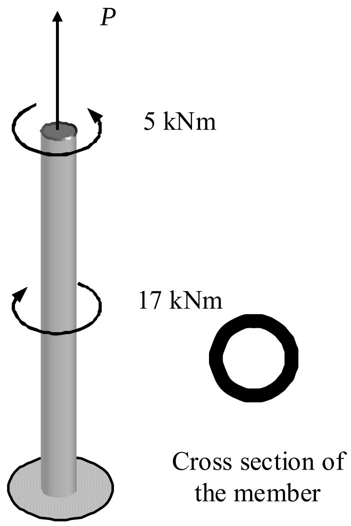
(a) **What is the self-weight of the beam per unit length?**

(b) **Determine the deflection of the beam at a distance of 2*L*/3 along the beam**, firstly as a single reduced general algebraic expression in terms of *w*, *I*, *L*, and *E*, and secondly as a numerical value by substituting the appropriate values for those quantities.

(c) The steel beam is replaced by another beam of identical dimensions but made of a material with half the density of steel and half the elastic modulus of steel. **How will the deflections and bending stresses due to the self-weight be affected. Briefly explain the answer**.

Part B – Applied Questions

4) 20 Marks



A 165.1 × 3.0 CHS manufactured by Smorgon Steel Tube Mills is built-in at the base and is experiencing a variety of loads. It experiences 2 applied twisting moments and an unknown axial tension. The torques are applied at mid-height and at the end, while the tension is applied at the free end. At the support, there is a point on the outer surface of the CHS which is under consideration.

The Smorgon Steel Tube Mills product catalogue gives the following geometric properties for this section $A = 1530$ mm^2, $I = 5.02 \times 10^6$ mm^4, $Z = 60.8 \times 10^3$ mm^3, $J = 10.0 \times 10^6$ mm^4.

(a) Draw the BMD, TMD and AFD and determine the internal actions that apply at the cross-section at the support.

(b) At the point under consideration, **determine the stress due to the torsion. Is this a normal stress or a shear stress**?

(c) Use von Mises' yield criterion and given that $f_y = 400$ MPa, **determine the maximum value of *P* that will cause yielding** due to the combined stresses caused by *P* and the applied torques.

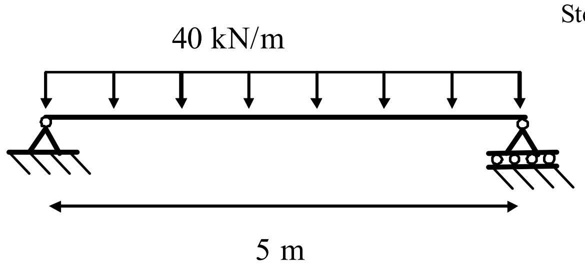
(d) **Draw a diagram that shows the stresses acting at this point, draw the Mohr's circle, and establish the principal stresses** for this point based on the value of *P* calculated in (c).

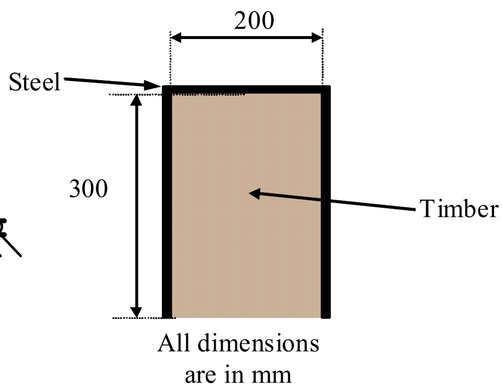
(e) **How would the answers to (b) and (c) above change** if the point under consideration was on the inside surface of the CHS? **Explain the answer.**

5) 20 Marks

A builder strengthens a 200 mm × 300 mm timber beam by connecting three 4 mm steel plates to the top and the two sides of the beam as shown below (there is no steel plate on the bottom). The beam acts compositely. It should be assumed that E_{timber} = 10000 MPa and E_{steel} = 200000 MPa. The beam has a total length of 5 m. The beam is subjected to a UDL of 40 kN/m.

a) **Draw the bending moment diagram**, and clearly identify the locations and magnitudes of the maximum bending moment.
b) Consider the cross section of the beam that experiences the largest absolute value of moment. **Draw the strain and stress distributions** due to bending at this section.
c) The builder now has the option of either (i) adding a 4th steel plate on the bottom side of the beam, or (ii) making the top plate twice as thick (8 mm), leaving the bottom side unreinforced. **Which option would be a better use of material? Explain the answer.**





Typical Beam Cross-Section

6) 20 Marks

A concrete column, height 8 m, with square cross-section 350 mm × 350 mm is symmetrically reinforced with 9 steel reinforcing bars, each of diameter 20 mm. The centre of each corner reinforcing bar is located 60 mm from closest edges of the concrete beam. It can be assumed that $E_{concrete}$ = 30 GPa and E_{steel} = 200 GPa and that **both materials exhibit linear elastic behaviour**. The column experiences a concentric axial compression force.

The design engineer wants to ensure that the stress in the steel does not exceed $f_{steel,\ max}$ = 300 MPa and that the stress in the concrete does not exceed $f_{conc,\ max}$ = 40 MPa.

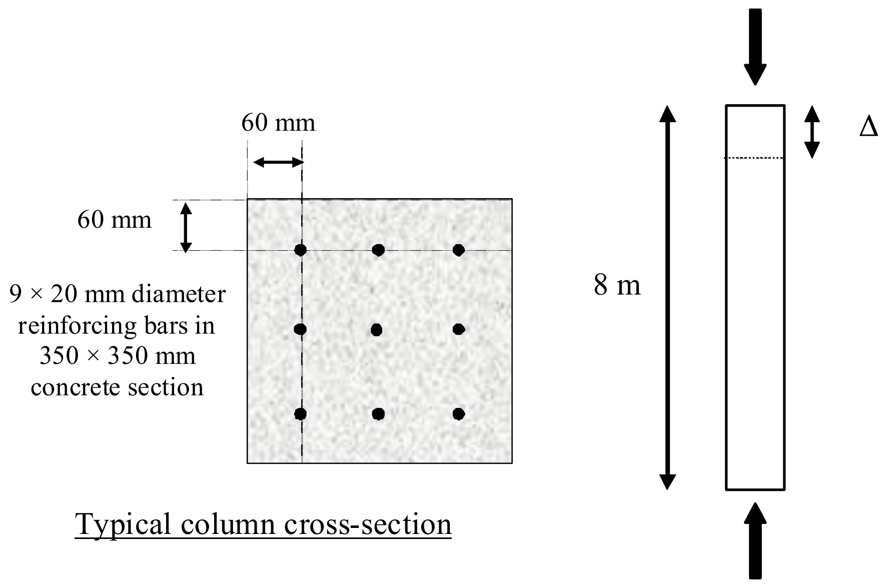
a) As the load increases from zero, **which material, steel or concrete, reaches its maximum permitted stress first? At what value of axial compression force does this occur?**

b) **For the value of load calculated in part (a) above, give the following:**

- Steel strain
- Steel stress
- Concrete strain
- Concrete stress
- Total axial shortening

c) The 20 mm diameter steel bars are replaced with 16 mm diameter bars. The concrete is not changed (it is reasonable to ignore the slight increase in concrete area). Assuming that the same load as calculated in (a) above is applied, **would the following increase or decrease (compared to the original case)? Explain your answers.**

- Steel strain
- Steel stress
- Concrete strain
- Concrete stress
- Total axial shortening



Typical column cross-section

Part C – Advanced Questions

The questions in this section are designed for students who seek to obtain a mark of Distinction or higher. They are not necessarily mathematically difficult, but require more thinking, visualisation, and application than the other questions. The majority of marks will be awarded to completed questions, and only minor marks will be awarded to partially answered questions – ie it is better to complete one question fully, than half complete 2 questions. The marks awarded in this section are bonus marks.

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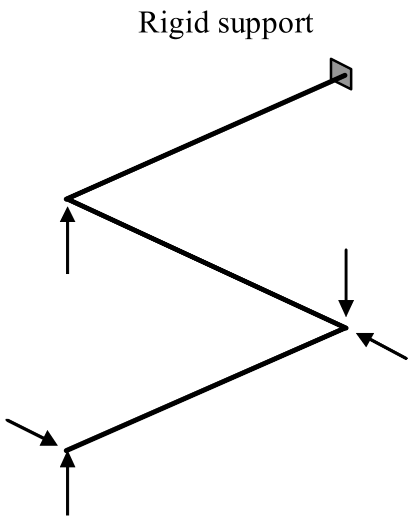
7) 10 Marks

The structure below is made up of three perpendicular components, each of length *L*. The structure lies in a horizontal plane. There are three vertical loads and two horizontal loads, each of magnitude *P*, acting as shown. The support is completely fixed, preventing deflection in every direction, and rotation about any axis. **Draw the bending moment, twisting moment, shear force, and axial force diagrams for the structure.**



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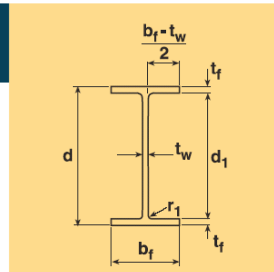
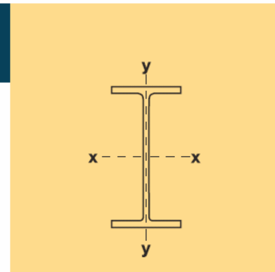
8) 10 Marks

Engineers have discovered a new material in which the elastic modulus in tension is 50000 MPa while the elastic modulus in compression is 200000 MPa. This material is used to make a beam with square cross-sectional dimensions of as yet unknown size. Consider a cross section that is resisting a positive bending moment of 600 kNm.

- To the next biggest 50 mm, determine the cross-sectional size of the beam to ensure that the stress in the material does not exceed 100 MPa (either in tension or compression)
- Draw the strain and stress distributions on the cross section (values).

This is the end of the examination paper, but there is an additional data sheet on page 8.





Universal Beams

Table 15 Universal Beams — Dimensions and Properties

Designation	Depth of Section d	Flange Width b_f	Flange Thickness t_f	Web Thickness t_w	Root Radius r_1	Depth Between Flanges d_1	$\frac{d_1}{t_w}$	$\frac{(b_f-t_w)}{2t_f}$	Gross Area of Cross-Section A_g	About x-axis I_x	Z_x	S_x	r_x	About y-axis I_y	Z_y	S_y	r_y	Torsion Constant J	Warping Constant I_w	Designation
kg/m	mm	mm	mm	mm	mm	mm			mm^2	10^6mm^4	10^3mm^3	10^3mm^3	mm	10^6mm^4	10^3mm^3	10^3mm^3	mm	10^3mm^4	10^9mm^6	
610 UB 125	612	229	19.6	11.9	14.0	572	48.1	5.54	16000	986	3230	3680	249	39.3	343	536	49.6	1560	3450	610 UB 125
113	607	228	17.3	11.2	14.0	572	51.1	6.27	14500	875	2880	3290	246	34.3	300	469	48.7	1140	2980	113
101	602	228	14.8	10.6	14.0	572	54.0	7.34	13000	761	2530	2900	242	29.3	257	402	47.5	790	2530	101
530 UB 92.4	533	209	15.6	10.2	14.0	502	49.2	6.37	11800	554	2080	2370	217	23.8	228	355	44.9	775	1590	530 UB 92.4
82.0	528	209	13.2	9.6	14.0	502	52.3	7.55	10500	477	1810	2070	213	20.1	193	301	43.8	526	1330	82.0
460 UB 82.1	460	191	16.0	9.9	11.4	428	43.3	5.66	10500	372	1610	1840	188	18.6	195	303	42.2	701	919	460 UB 82.1
74.6	457	190	14.5	9.1	11.4	428	47.1	6.24	9520	335	1460	1660	188	16.6	175	271	41.8	530	815	74.6
67.1	454	190	12.7	8.5	11.4	428	50.4	7.15	8580	296	1300	1480	186	14.5	153	238	41.2	378	708	67.1
410 UB 59.7	406	178	12.8	7.8	11.4	381	48.8	6.65	7640	216	1060	1200	168	12.1	135	209	39.7	337	467	410 UB 59.7
53.7	403	178	10.9	7.6	11.4	381	50.1	7.82	6890	188	933	1060	165	10.3	115	179	38.6	234	394	53.7
360 UB 56.7	359	172	13.0	8.0	11.4	333	41.6	6.31	7240	161	899	1010	149	11.0	128	198	39.0	338	330	360 UB 56.7
50.7	356	171	11.5	7.3	11.4	333	45.6	7.12	6470	142	798	897	148	9.60	112	173	38.5	241	284	50.7
44.7	352	171	9.7	6.9	11.4	333	48.2	8.46	5720	121	689	777	146	8.10	94.7	146	37.6	161	237	44.7
310 UB 46.2	307	166	11.8	6.7	11.4	284	42.3	6.75	5930	100	654	729	130	9.01	109	166	39.0	233	197	310 UB 46.2
40.4	304	165	10.2	6.1	11.4	284	46.5	7.79	5210	86.4	569	633	129	7.65	92.7	142	38.3	157	165	40.4
32.0	298	149	8.0	5.5	13.0	282	51.3	8.97	4080	63.2	424	475	124	4.42	59.3	91.8	32.9	86.5	92.9	32.0
250 UB 37.3	256	146	10.9	6.4	8.9	234	36.6	6.40	4750	55.7	435	486	108	5.66	77.5	119	34.5	158	85.2	250 UB 37.3
31.4	252	146	8.6	6.1	8.9	234	38.4	8.13	4010	44.5	354	397	105	4.47	61.2	94.2	33.4	89.3	65.9	31.4
25.7	248	124	8.0	5.0	12.0	232	46.4	7.44	3270	35.4	285	319	104	2.55	41.1	63.6	27.9	67.4	36.7	25.7
200 UB 29.8	207	134	9.6	6.3	8.9	188	29.8	6.65	3820	29.1	281	316	87.3	3.86	57.5	88.4	31.8	105	37.6	200 UB 29.8
25.4	203	133	7.8	5.8	8.9	188	32.3	8.15	3230	23.6	232	260	85.4	3.06	46.1	70.9	30.8	62.7	29.2	25.4
22.3	202	133	7.0	5.0	8.9	188	37.5	9.14	2870	21.0	208	231	85.5	2.75	41.3	63.4	31.0	45.0	26.0	22.3
18.2	198	99	7.0	4.5	11.0	184	40.9	6.75	2320	15.8	160	180	82.6	1.14	23.0	35.7	22.1	38.6	10.4	18.2
180 UB 22.2	179	90	10.0	6.0	8.9	159	26.5	4.20	2820	15.3	171	195	73.6	1.22	27.1	42.3	20.8	81.6	8.71	180 UB 22.2
18.1	175	90	8.0	5.0	8.9	159	31.8	5.31	2300	12.1	139	157	72.6	0.975	21.7	33.7	20.6	44.8	6.80	18.1
16.1	173	90	7.0	4.5	8.9	159	35.3	6.11	2040	10.6	123	138	72.0	0.853	19.0	29.4	20.4	31.5	5.88	16.1
150 UB 18.0	155	75	9.5	6.0	8.0	136	22.7	3.63	2300	9.05	117	135	62.8	0.672	17.9	28.2	17.1	60.5	3.56	150 UB 18.0
14.0	150	75	7.0	5.0	8.0	136	27.2	5.00	1780	6.66	88.8	102	61.1	0.495	13.2	20.8	16.6	28.1	2.53	14.0