The University of Sydney

School of Civil Engineering

CIVL2201 Structural Mechanics

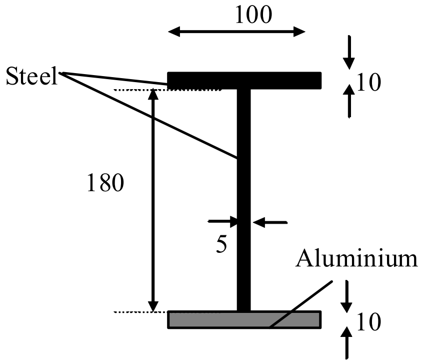
Assignment 2 – Bending and The World Trade Center

This assignment is to be submitted by 2 pm Thursday 21 May 2009. Submissions should be made directly to your tutor. Please submit stapled sheets only (no manilla folders or slip in sheet protectors etc).

This assignment is worth 5 % of the total mark.

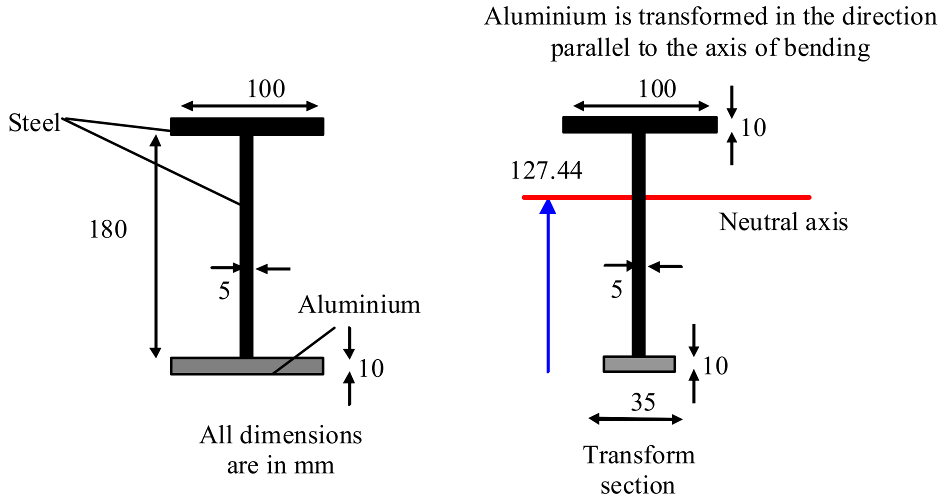
Please attach the "Assignment Cover Sheet", available from the Structural Mechanics website, as the first page of the submission.

1) A company has manufactured a new type of doubly symmetric composite I section from steel and aluminium plates, the cross-section of which is shown. It can be assumed that $E_{\text{aluminium}} = 70000$ MPa and $E_{\text{steel}} = 200000$ MPa, and that both materials exhibit linear elastic behaviour.
   a) Consider a cross-section of the beam which is subjected to a positive bending moment of 50 kNm about its horizontal centroidal axis.
      i. Determine the curvature
      ii. Draw the strain distribution across the section (values required).
      iii. Draw the stress distribution across the section (values required).



All dimensions are in mm

The first step is to produce the *transform section* by establishing the modular ratio, $n = E_{al} / E_{st} = 70000/200000 = 0.35$. The aluminium can be transformed to steel by multiplying its width by that factor. (The beam is bent about the horizontal axis, so only horizontal dimensions should be transformed).



Location of centroid from bottom:

$y_{centroid} = (35\times10\times5 + 180\times5\times(90+10) + 100\times10\times195)/(35\times10 + 180\times5 + 100\times10) = 127.44$ mm

Note that the NA is located above the midheight of the section – this is logical as the top flange is much than the bottom – so the centroid should be closer to the top. There is no need to establish the horizontal position of the centroid as this question only involves bending about the horizontal axis, but it obvious that it lies on the vertical axis of symmetry.

Second moment of area – use parallel axis theorem:

$$I_x = 35\times10^3/12 + 35\times10\times(127.44-5)^2 + 180^3\times5/12 + 180\times5\times(127.44-100)^2 + 100\times10^3/12 + 100\times10\times(127.44-195)^2$$
$$= 12.96 \times 10^6 \text{ mm}^4$$

(If you had transformed to aluminium $I = 36.94 \times 10^6$ mm^4)

Determine curvature:

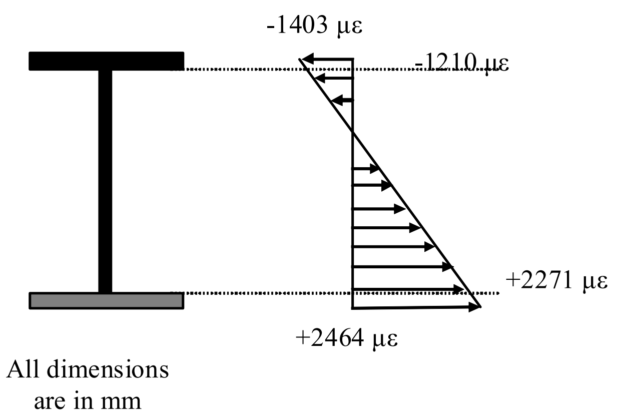
$\kappa = M/EI = 50000000$ Nmm / (200000 MPa $\times$ 12.93 $\times$ 10^6 mm^4) = 19.33 $\times$ 10^{-6} mm^{-1}

(Note that since the section is now transformed to steel, curvature is determined using E_{st}).

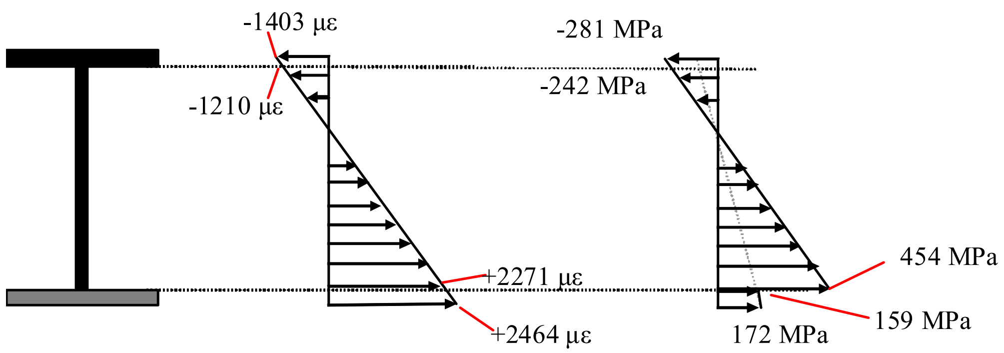
The strain at any height from the neutral axis (centroid) can be determined as $\varepsilon = \kappa y$

The strain distribution can thus be determined. The bottom flange is 127.44 mm from the centroid and hence the strain at that point is $127.44\times19.33\times10^{-6}$ = 2464 με, and the top flange is 72.56 mm from the centroid and hence the strain at that point is $72.56\times19.33\times10^{-6}$ = 1403 με. Note that the strain distribution is linear, regardless of the fact that there are two different materials. Note that the applied moment was a positive moment, and hence the beam is sagging and it is compression at the top.

Using similar equations the strains at the top and bottom of the web (at the flange/web interface) are -1210 με and +2271 με respectively.



The stresses at any point can be determined as $f = My/I$ but note carefully that this assumes that all the material is steel. The equation $f = My/I$ applies to steel, but $f = nMy/I$ applies to the aluminium. Alternatively, one can multiply the strains by the appropriate elastic modulus (either E_{steel} or E_{al}) to get the stresses.



Note that the strain is continuous and linear (that is what composite action means) but the discontinuity of material properties causes a discontinuity in stress. Note that if a line is drawn between the two stress distributions that it crosses at the neutral axis. Also note the discontinuity of the stresses at the steel/aluminium interface represents a factor of 0.35, which corresponds to the modular ratio.

A pole is made from 2 rigidly connected circular hollow sections each 3 m in length. The pole is loaded by two twisting moments, 40 kNm and 20 kNm (in the same direction) at heights of 3 m and 6 m respectively (measured from the base). Each section of the pole has an outer diameter of 300 mm, but the material varies. The bottom half is made from steel (G_{steel} = 80000 MPa) with thickness 4 mm, and the top half is made from aluminium ($G_{aluminium}$ = 40000 MPa) with an unknown thickness. The pole is rigidly connected to a support at the base, and free at the top.

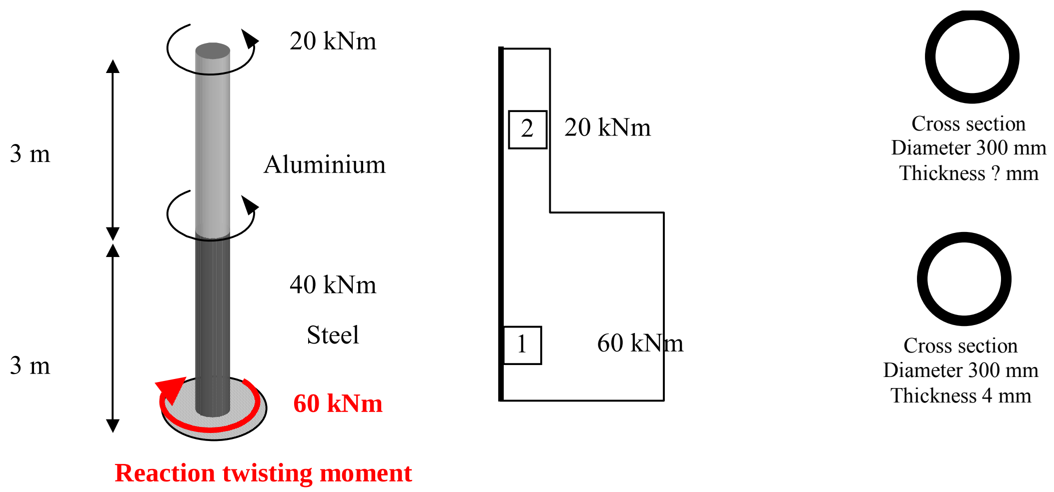
(a) Draw the FBD & twisting moment diagram.

(b) Determine the thickness of the aluminium portion in order to ensure that the maximum shear stress in the aluminium is one half of the maximum shear stress in the steel. What is the maximum shear stress in both the steel and aluminium?

(c) Draw a diagram that shows the rotation with respect to the fixed base at any height.

(d) Would the total twist rotation at the top relative to the fixed base increase or decrease if the materials were swapped (steel in the top half, aluminium in the bottom)? Briefly explain your answer.

The first step is to draw a diagram, from which the TMD can be obtained by inspection:



The values will be determined for each "zone" of the pole – either 1 or 2.

The cross-section for the steel is known.

$$J_{st} = \frac{\pi D^4_{outer} - \pi D^4_{inner}}{32} = \frac{\pi\, 300^4 - \pi\, 292^4}{32} = 81.49 \times 10^6 \text{ mm}^4$$

The shear stress at any point is given by $\tau = Tr/J$. So the maximum stress in the steel is given by:

$$\tau_{steel} = \frac{T\,R}{J} = \frac{60 \text{ kNm} \times 300/2 \text{ mm}}{81.49 \times 10^6 \text{ mm}^4} = 110.4 \text{ MPa}$$

Hence the shear stress in the aluminium must be 55.2 MPa. Hence

$$\tau_{al} = \frac{T\ R}{J_{al}} = \frac{20\ \text{kNm} \times 300/2\ \text{mm}}{J_{al}} = 55.2\ \text{MPa}$$

$$J_{al} = 54.33 \times 10^6\ \text{mm}^4 = \frac{\pi\ D_{outer}^4 - \pi\ D_{inner}^4}{32}$$

$D_{inner,\ al}$ = 294.74 mm, hence t_{al} = 2.63 mm.

Rate of twist should be determined for each zone:

$$\left(\frac{d\phi}{dz}\right)_1 = \frac{T}{G\ J} = \frac{60\ \text{kNm}}{80000\ \text{MPa} \times 81.49 \times 10^6\ \text{mm}^4} = 9.203 \times 10^{-6}\ \text{rads} \cdot \text{mm}^{-1}$$

$$\left(\frac{d\phi}{dz}\right)_2 = \frac{T}{G\ J} = \frac{20\ \text{kNm}}{40000\ \text{MPa} \times 54.33 \times 10^6\ \text{mm}^4} = 9.203 \times 10^{-6}\ \text{rads} \cdot \text{mm}^{-1}$$

Note that it was pure luck (and not my intention) that the rates of twist in the steel and aluminium ended up the same. This would not normally happen.

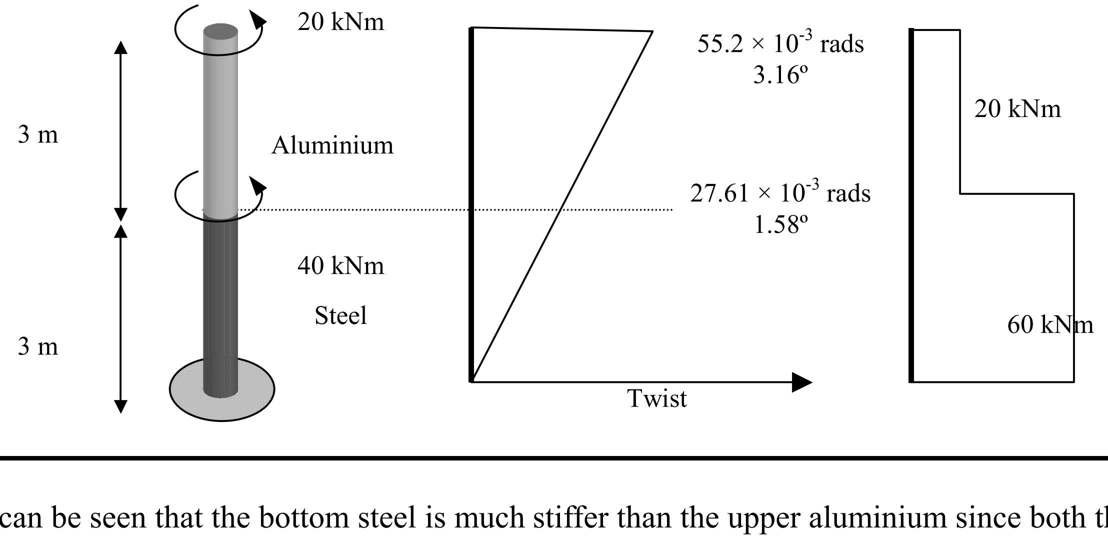
Since the torque is constant then the rate of twist is constant in each zone, and hence the total twist in each zone (relative to the bottom of the zone) is just $\phi = \frac{d\phi}{dz} \cdot L$.

$$\phi_1 = 9.203 \times 10^{-6}\ \text{rads} \cdot \text{mm}^{-1} \times 3000\ \text{mm} = 27.61 \times 10^{-3}\ \text{rads} = 1.58°$$

$$\phi_2 = 9.203 \times 10^{-6}\ \text{rads} \cdot \text{mm}^{-1} \times 3000\ \text{mm} = 27.61 \times 10^{-3}\ \text{rads} = 1.58°$$

The twist is *cumulative* up the length of the pole (note the torques were all in the same direction – so the twists add to each other). In each zone the torque and the rate of change of twist are constant, so the twist v height curve should be linear.

At 3 m the twist is 27.61×10^{-3} rads.
At 6 m the twist is $227.61 \times 10^{-3} + (27.61 \times 10^{-3}) = 55.2 \times 10^{-3}$ rads.



It can be seen that the bottom steel is much stiffer than the upper aluminium since both the *G* and *J* values are higher. Even though the torque on the steel is 3 times that on the aluminium, the twist is the same due to the extra stiffness. If the two halves are swapped, the aluminium twist will increase 3 fold (big increase), while the steel twist will reduce to one third (a smallish reduction), so the overall effect is that the total twist will increase.

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3) Read all of Chapter 2 of the ASCE / FEMA *World Trade Center Building Performance Study* which is on the internet at http://www.civil.usyd.edu.au/courses/civl2201/. Based on your student number, choose a portion of the allocated section of the report and in approximately 100 – 200 words plus a diagram explain what you have learned from reading this particular section. Directly quoting from the report would not be deemed suitable. (Students are not expected to summarise the entire allocated section in 100 words, just a small portion of their choice). Also if students want to summarise another different part of the report (but not any of those listed below) in 100 – 200 words, that is acceptable.

Second last number of student number: 9: External columns (pp 2.2 – 2.4)
Second last number of student number: 8: Composite floor/deck (pp 2.3 – 2.5)
Second last number of student number: 7: Outrigger truss (pp 2.5 – 2.10)
Second last number of student number: 6: Basement/foundations (pp 2.10 – 2.11)
Second last number of student number: 5: Passive fire systems (p 2.12)
Second last number of student number: 4: Fire suppression systems (pp 2.12 – 2.13)
Second last number of student number: 3: WTC 1 – Impact damage (pp 2.15 – 2.21)
Second last number of student number: 2: WTC 1 – Fire development (pp 2.21 – 2.23)
Second last number of student number: 1: WTC 1 – Structural response to fire (pp 2.24 – 2.26)
Second last number of student number: 0: WTC 1 - Collapse (pp 2.27)

Did you remember the correct units?

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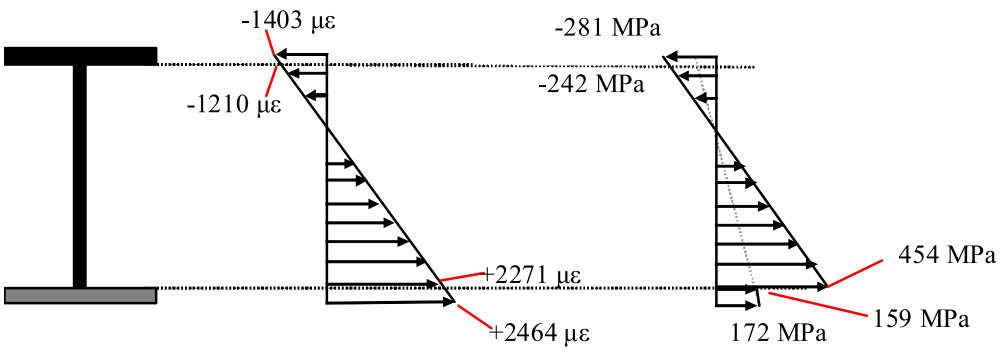
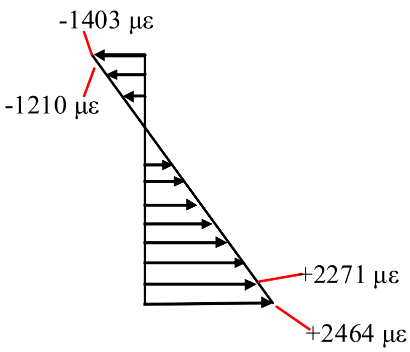
Comments

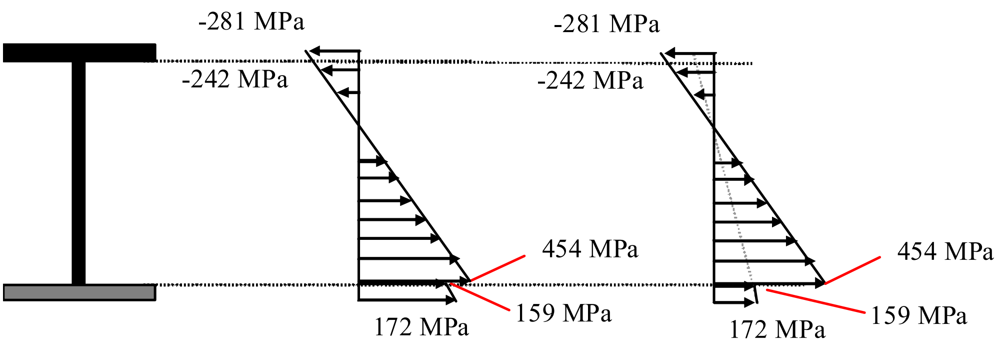
- **Mean: 3.85/5, median 4/5.**
- This was very much a mixed assignment, with some students putting in a very good effort, and other students clearly putting in very little and very rushed effort at the last minute.
- Note that only Question 1 was marked in detail – students should look at the solutions to Q2.
- **Diagrams, Diagrams, Diagrams.** Unfortunately there was very poor use of diagrams in this assignment and this is reflected in the marks.
  - It is **impossible** to do a question about bending and stress without drawing the beam itself, and both the original and transform cross-section. Some students had not a single diagram – how could any reasonable observer have any idea what you are doing if there are no diagrams. This is below the standard we expect and you will have great difficulty passing any course or being a good engineer if you do not draw decent diagrams.
  - To make it easy for someone to interpret the results, draw the section, the strain diagram and the stress diagram all next to each other horizontally – see the solution (and next page).
- **Layout:**
  - Many students may have received a comment such as "draw stress and strain next to each other". What does this mean? Consider the following diagrams ….



  - In this diagram, the section, stress picture and strain picture are all together, next to each other. It is extremely clear what is the value of the stress and strain at any point. The answers are communicated quickly, simply and clearly.



  - Many students gave separate diagrams – such as the one above. Quite often the stress and strain pictures were on different pages, and there was no diagram of the I-section next to the picture. Which diagram more appropriately communicates the answers? Which diagram belongs to a distinction or high distinction submission as opposed to a credit/pass assignment?



- **Scale:** Which of the above stress diagrams better indicates the scale of the stresses. At the steel/aluminium the stress jumps from 454 MPa to 159 MPa – this is a big discontinuity where the stress falls to 35% of the steel value due to the modular ratio between steel and concrete. However when you look at the stress picture on the left the diagram, the discontinuity in minor, and the diagram suggests that the steel and aluminium stresses are roughly the same. The diagram on the right does give a much better representation of the relative sizes of the stresses. In your answers you don't have to have the diagrams perfectly to scale, but they should be reasonable. It is this type of care with detail and understanding of results that makes the difference between pass/credit and distinction/high distinction.

- **Question 3**: Please read the question – a diagram was required.

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