The University of Sydney

School of Civil Engineering

CIVL2201 Structural Mechanics

Semester 1, 2008

Time Allowed: 3 hours + 20 minutes reading time

Instructions to Candidates

(a) This paper contains 3 parts - A, B, & C; and is marked out of 90.

- **Part A has 3 questions (Q 1 – Q 3, 30 marks).**
- **Part B has 3 questions (Q 4 – Q 6, 60 marks).**
- **Part C has 2 questions (Q 7 - Q 8, 20 marks - which are bonus marks).**

(b) All questions may be attempted.

(c) Suitable working, diagrams and explanations are required for each question.

(d) Marks may be deducted for work that is not satisfactorily set out.

(e) Units are important.

(f) Programmable and non-programmable calculators may be used.

(g) Read the questions carefully before answering.

(h) **Annotated copies of the "Structural Mechanics" Text Book or Lecture Notes may be taken into the exam, but no other written material is permitted.** Other sheets of paper may not be inserted into the set of lecture notes.

(i) Each question has an "explain" type component. It is expected that students should answer these parts with a short written explanation (one or two paragraphs) plus a diagram. Performing calculations is not necessarily expected, but using equations might also be useful.

Part A – Fundamental Questions

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1) 10 Marks

Consider a cantilever of length 8 m where the fixed support is at the left hand end of the beam. There is a uniformly distributed load (UDL) of 2 N/mm, acting vertically downwards between the mid point and the free end of the cantilever (ie the UDL acts over a length of 4 m), and there is a point load of 5 kN acting vertically downwards at the mid point. The effects of self-weight are negligible compared to the effects of the applied loading. The beam is made from steel which has an elastic modulus of 200000 MPa.

(a) **Draw the free body diagram** of the beam clearly showing the support reactions. **Calculate** the values of the support reactions.

(b) **Draw the bending moment diagram (BMD) and shear force diagram (SFD). Clearly indicate the values** of bending moment and shear force at the key points along the beam (the supports, midpoint, free end and the location of the maximum bending moment).

(c) It is generally assumed that steel is *linear elastic*. With the aid of diagrams, and by specifically referring to the results of Lab Session 2, **briefly explain what *linear elastic* behaviour means with respect to a steel beam under load**.

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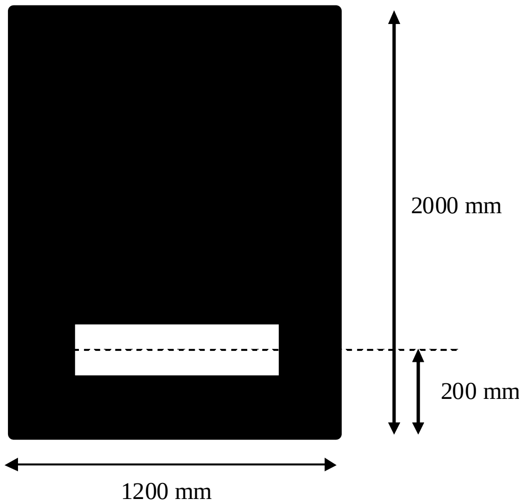
2) 10 Marks

A large rectangular concrete beam has cross-sectional dimensions of 1200 mm × 2000 mm as shown. Running through the cross-section is a rectangular void, 800 × 200 mm, where the centre of the void is 200 mm from the bottom of the cross section. The section is symmetric about the vertical axis. The cross section is made from concrete which should be assumed to be linear elastic with an elastic modulus of 25000 MPa.



(a) **Calculate the location of the centroid and show this location** on a diagram.

(b) **Calculate the second moments of area** (I_x & I_y) of the section about the horizontal and vertical centroidal axes.

(c) The design engineer wants to maximise the values of I_x & I_y by moving the internal void to a different location in the cross-section (though the void must remain as a 800 mm × 200 mm rectangle). **Identify the optimal position of the void** (showing it on a diagram). **Briefly justify the answer.** Calculations are not required, but identifying relevant equations would help the answer.

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3) 10 marks

In Example 9.2.3 of the lecture notes, it is shown that the deflection of a simply supported beam with a UDL is given by $v = \frac{w}{24EI}\left(z^4 - 2Lz^3 + L^3 z\right)$.

Consider a 460UB82.1 steel beam, with a span of 8 m, under its self-weight only. The relevant page of the section properties handbook is given on the back page on this examination sheet – students are free to use the values given in that table without the need to recalculate them. The density of steel is 7850 kg/m^3 and the elastic modulus is E = 200000 MPa. Linear elastic material properties should be assumed. The beam is oriented so that its web is in the vertical plane. Gravity may be assumed as 10 m/s^2.

(a) **What is the self-weight of the beam per unit length?**
(b) **Determine the deflection of the beam at a distance of 3*L*/4 along the beam**, firstly as a single reduced general algebraic expression in terms of *w*, *I*, *L*, and *E*, and secondly as a numerical value by substituting the appropriate values for those quantities.
(c) The designer wishes to decrease the deflection of beam, and suggests stiffening the beam by welding steel plates on each side of the web of the I-section along its length. **Comment on the effectiveness of this approach, and, if appropriate, suggest a more effective method**. Calculations are not required, though referring to equations and diagrams would be beneficial to the answer.

Part B – Applied Questions

4) 20 Marks

Consider a beam made from 610UB101 steel section which is oriented so that the web is in the vertical plane. A given cross section is subjected to a sagging bending moment of 700 kNm about the horizontal axis, and a vertical shear force of 600 kN. The material has linear elastic material properties. Students may use any geometric properties that are listed in the table at the end of the exam paper without calculations.

a) **Draw the stress distribution in the section due to the bending moment** (values not required). **Draw the stress distribution in the section due to the shear force** (values not required). **Explain the difference between the types of stresses** resulting from the bending moment and the shear force.

b) Consider a point at the top of the web, just below the flange web junction. **Determine all the stresses at that point, and draw a diagram to represent the stresses. Draw the Mohr's circle that represents the stresses. What are the principal stresses**?

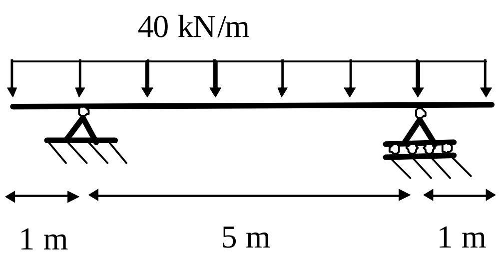
c) Assuming the section is made from a material with yield stress f_y = 300 MPa, **determine whether the material has yielded according to both Tresca's and von Mises' yield criteria.**

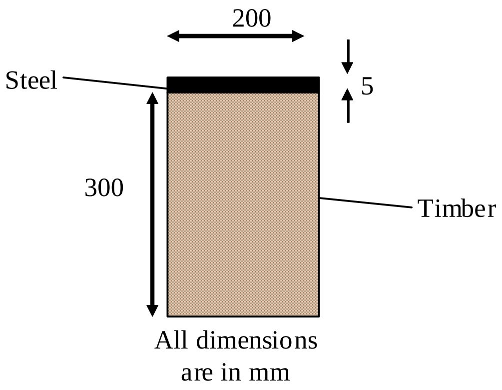
d) Without performing any calculations, **comment on how the answers to b) and c) above might change** if the point under consideration was in the top flange just above the flange web junction.

5) 20 Marks

A builder strengthens a 200 mm × 300 mm timber beam by connecting 5 mm steel plate to the top of the beam as shown below. The beam acts compositely. It should be assumed that E_{timber} = 10000 MPa and E_{steel} = 200000 MPa. The beam has a total length of 7 m, with a central span of 5 m and an overhang of 1 m at each end, as shown below. The beam is subjected to a UDL of 40 kN/m.

a) **Draw the bending moment diagram**, and clearly identify the locations and magnitudes of the maximum sagging and hogging bending moments.
b) Consider the cross section of the beam that experiences the largest absolute value of moment (this could be either hogging or sagging). **Draw the strain and stress distributions** due to bending at this section.
c) The steel plate is replaced by aluminium, which has an elastic modulus of 70000 MPa. **How will the BMD, the location of the neutral axis, and the deflection of the beam change? Explain the answer.**





Typical Beam Cross-Section

6) 20 Marks

An innovation in structural engineering is the Concrete Filled Double Skinned Tube (CFDST). A CFDST consists of a hollow section placed inside another larger hollow section, and the space in between filled with concrete. The inside of the inner tube is not filled. Once filled, the CFDST acts compositely.

The CFDST has the following properties:

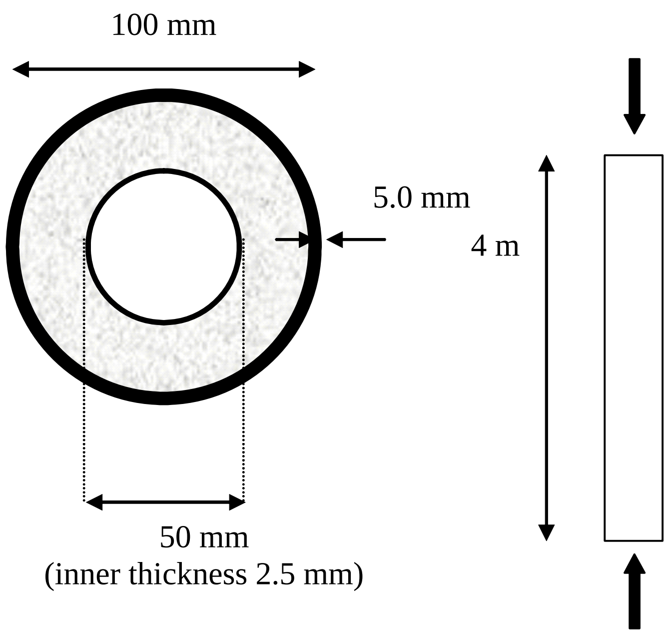
- Steel outer tube with dimensions 100 mm × 5 mm, steel inner tube with dimensions 50 mm × 2.5 mm, and E_{steel} = 200000 MPa.
- Concrete infill, with $E_{concrete}$ = 40000 MPa.
- All materials behave linear elastically.
- The member is used as a column, 4 m in length, and is subjected to an axial compression.
- The design engineer wants to ensure that the stresses do not exceed $f_{concrete, max}$ = 25 MPa, or $f_{steel, max}$ = 250 MPa.

a) As the axial compression increases, **which material will reach its maximum permissible stress first –** the steel or the concrete? **Calculate the total compression load** at which this occurs.

b) At the load calculated in (a) above, give values for the following:

- **Concrete strain and stress**
- **Steel strain and stress**
- **Total axial shortening**

c) For this column to achieve maximum efficiency, it would have been desirable for both the steel and concrete to reach their maximum permissible stress at the same time. This has not occurred (as per the answer in part (a)). **Should the engineer either increase, decrease (or do nothing to) the thickness of the inner tube to try to improve the efficiency of the column**? Briefly explain the answer (calculations not required).



Typical column cross-section

Part C – Advanced Questions

The questions in this section are designed for students who seek to obtain a mark of Distinction or higher. They are not necessarily mathematically difficult, but require more thinking, visualisation, and application than the other questions. The majority of marks will be awarded to completed questions, and only minor marks will be awarded to partially answered questions – ie it is better to complete one question fully, than half complete 2 questions. The marks awarded in this section are bonus marks.

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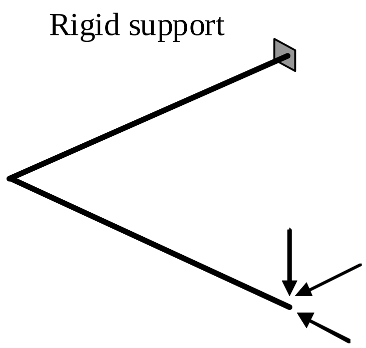
7) 10 Marks

The structure below is made up of two perpendicular components, each of length L. The structure lies in a horizontal plane. There are three perpendicular loads, each of magnitude P, acting at the free end of the structure. The support is completely fixed, preventing deflection in every direction, and rotation about any axis. **Draw the bending moment, twisting moment, shear force and axial force diagrams for the structure.** Note that 2 BMDs and 2 SFDs will be required – explain why.



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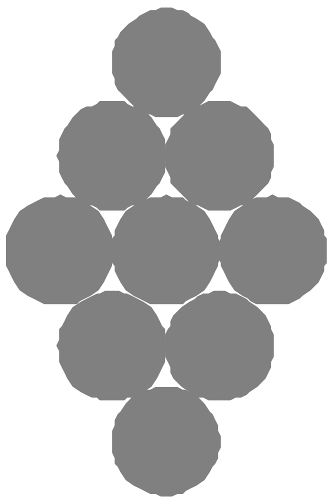
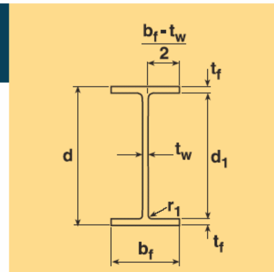
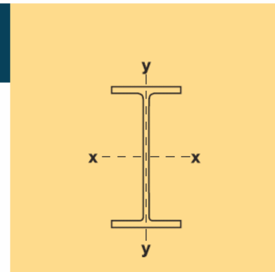
8) 10 Marks

An engineer creates a new beam by placing together nine identical solid circular rods so that they just touch and welds them together along their junctions to ensure that they act compositely. The typical cross-section is shown. Assuming each rod has diameter D, **determine the second moment of area** about the horizontal centroidal axis of the new hybrid section in terms of D.



This is the end of the examination paper, but there is an additional data sheet on page 8.





Universal Beams

Table 15 Universal Beams — Dimensions and Properties

Designation	Depth of Section d	Flange Width b_f	Flange Thickness t_f	Web Thickness t_w	Root Radius r_1	Depth Between Flanges d_1	$\frac{d_1}{t_w}$	$\frac{(b_f-t_w)}{2t_f}$	Gross Area of Cross-Section A_g	About x-axis I_x	Z_x	S_x	r_x	About y-axis I_y	Z_y	S_y	r_y	Torsion Constant J	Warping Constant I_w	Designation
kg/m	mm	mm	mm	mm	mm	mm			mm^2	10^6mm^4	10^3mm^3	10^3mm^3	mm	10^6mm^4	10^3mm^3	10^3mm^3	mm	10^3mm^4	10^9mm^6	
610 UB 125	612	229	19.6	11.9	14.0	572	48.1	5.54	16000	986	3230	3680	249	39.3	343	536	49.6	1560	3450	610 UB 125
113	607	228	17.3	11.2	14.0	572	51.1	6.27	14500	875	2880	3290	246	34.3	300	469	48.7	1140	2980	113
101	602	228	14.8	10.6	14.0	572	54.0	7.34	13000	761	2530	2900	242	29.3	257	402	47.5	790	2530	101
530 UB 92.4	533	209	15.6	10.2	14.0	502	49.2	6.37	11800	554	2080	2370	217	23.8	228	355	44.9	775	1590	530 UB 92.4
82.0	528	209	13.2	9.6	14.0	502	52.3	7.55	10500	477	1810	2070	213	20.1	193	301	43.8	526	1330	82.0
460 UB 82.1	460	191	16.0	9.9	11.4	428	43.3	5.66	10500	372	1610	1840	188	18.6	195	303	42.2	701	919	460 UB 82.1
74.6	457	190	14.5	9.1	11.4	428	47.1	6.24	9520	335	1460	1660	188	16.6	175	271	41.8	530	815	74.6
67.1	454	190	12.7	8.5	11.4	428	50.4	7.15	8580	296	1300	1480	186	14.5	153	238	41.2	378	708	67.1
410 UB 59.7	406	178	12.8	7.8	11.4	381	48.8	6.65	7640	216	1060	1200	168	12.1	135	209	39.7	337	467	410 UB 59.7
53.7	403	178	10.9	7.6	11.4	381	50.1	7.82	6890	188	933	1060	165	10.3	115	179	38.6	234	394	53.7
360 UB 56.7	359	172	13.0	8.0	11.4	333	41.6	6.31	7240	161	899	1010	149	11.0	128	198	39.0	338	330	360 UB 56.7
50.7	356	171	11.5	7.3	11.4	333	45.6	7.12	6470	142	798	897	148	9.60	112	173	38.5	241	284	50.7
44.7	352	171	9.7	6.9	11.4	333	48.2	8.46	5720	121	689	777	146	8.10	94.7	146	37.6	161	237	44.7
310 UB 46.2	307	166	11.8	6.7	11.4	284	42.3	6.75	5930	100	654	729	130	9.01	109	166	39.0	233	197	310 UB 46.2
40.4	304	165	10.2	6.1	11.4	284	46.5	7.79	5210	86.4	569	633	129	7.65	92.7	142	38.3	157	165	40.4
32.0	298	149	8.0	5.5	13.0	282	51.3	8.97	4080	63.2	424	475	124	4.42	59.3	91.8	32.9	86.5	92.9	32.0
250 UB 37.3	256	146	10.9	6.4	8.9	234	36.6	6.40	4750	55.7	435	486	108	5.66	77.5	119	34.5	158	85.2	250 UB 37.3
31.4	252	146	8.6	6.1	8.9	234	38.4	8.13	4010	44.5	354	397	105	4.47	61.2	94.2	33.4	89.3	65.9	31.4
25.7	248	124	8.0	5.0	12.0	232	46.4	7.44	3270	35.4	285	319	104	2.55	41.1	63.6	27.9	67.4	36.7	25.7
200 UB 29.8	207	134	9.6	6.3	8.9	188	29.8	6.65	3820	29.1	281	316	87.3	3.86	57.5	88.4	31.8	105	37.6	200 UB 29.8
25.4	203	133	7.8	5.8	8.9	188	32.3	8.15	3230	23.6	232	260	85.4	3.06	46.1	70.9	30.8	62.7	29.2	25.4
22.3	202	133	7.0	5.0	8.9	188	37.5	9.14	2870	21.0	208	231	85.5	2.75	41.3	63.4	31.0	45.0	26.0	22.3
18.2	198	99	7.0	4.5	11.0	184	40.9	6.75	2320	15.8	160	180	82.6	1.14	23.0	35.7	22.1	38.6	10.4	18.2
180 UB 22.2	179	90	10.0	6.0	8.9	159	26.5	4.20	2820	15.3	171	195	73.6	1.22	27.1	42.3	20.8	81.6	8.71	180 UB 22.2
18.1	175	90	8.0	5.0	8.9	159	31.8	5.31	2300	12.1	139	157	72.6	0.975	21.7	33.7	20.6	44.8	6.80	18.1
16.1	173	90	7.0	4.5	8.9	159	35.3	6.11	2040	10.6	123	138	72.0	0.853	19.0	29.4	20.4	31.5	5.88	16.1
150 UB 18.0	155	75	9.5	6.0	8.0	136	22.7	3.63	2300	9.05	117	135	62.8	0.672	17.9	28.2	17.1	60.5	3.56	150 UB 18.0
14.0	150	75	7.0	5.0	8.0	136	27.2	5.00	1780	6.66	88.8	102	61.1	0.495	13.2	20.8	16.6	28.1	2.53	14.0