The University of Sydney

School of Civil Engineering

CIVL2201 Structural Mechanics

Semester 1, 2007

Time Allowed: 3 hours + 20 minutes reading time

Instructions to Candidates

(a) This paper contains 3 parts - A, B, & C; and is marked out of 90.

- **Part A has 3 questions (Q 1 – Q 3, 30 marks).**
- **Part B has 3 questions (Q 4 – Q 6, 60 marks).**
- **Part C has 2 questions (Q 7 - Q 8, 20 marks - which are bonus marks).**

(b) All questions may be attempted.

(c) Suitable working, diagrams and explanations are required for each question.

(d) Marks may be deducted for work that is not satisfactorily set out.

(e) Units are important.

(f) Programmable and non-programmable calculators may be used.

(g) Read the questions carefully before answering.

(h) **Annotated copies of the "Structural Mechanics" Text Book or Lecture Notes may be taken into the exam, but no other written material is permitted.** Other sheets of paper may not be inserted into the set of lecture notes.

(i) Each question has an "explain" type component. It is expected that students should answer these parts with a short written explanation (one or two paragraphs) plus a diagram. Performing calculations is not necessarily expected, but using equations might also be useful.

Part A – Fundamental Questions

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1) 10 Marks

Consider a simply supported beam of length 6 m. There is a uniformly distributed load (UDL) of 2 N/mm, acting vertically downwards between the left hand support and a point 2 m from the right hand support (ie the UDL acts over a length of 4 m). The effects of self-weight are negligible compared to the effects of the applied loading. The beam is made from steel which has an elastic modulus of 200000 MPa.

(a) **Draw the free body diagram** of the beam clearly showing the support reactions. **Calculate** the values of the support reactions.

(b) **Draw the bending moment diagram (BMD) and shear force diagram (SFD). Clearly indicate the values** of bending moment and shear force at the key points along the beam (the supports and the location of the maximum bending moment).

(c) It is generally assumed that steel is *linear elastic*. With the aid of diagrams, and by specifically referring to the results of Lab Session 2, **briefly explain what *linear elastic* behaviour means with respect to a steel beam under load**.

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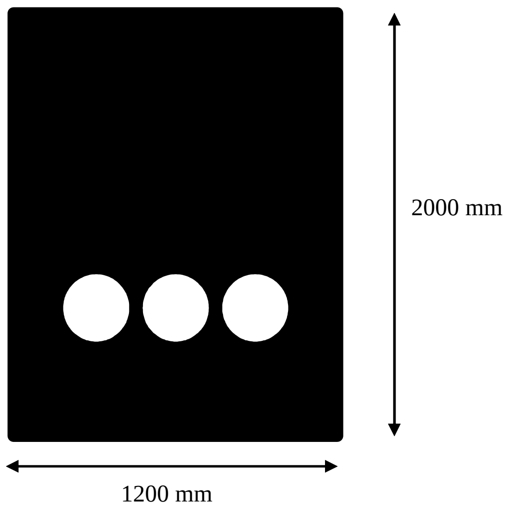
2) 10 Marks

A large concrete beam has cross-sectional dimensions of 1200 mm × 2000 mm as shown. Running through the cross-section are three circular voids, each of diameter 200 mm, where the centre of each void is 500 mm from the bottom of the cross section. The section is symmetric about the vertical axis. The cross section is made from concrete which should be assumed to be linear elastic with an elastic modulus of 25000 MPa.



(a) **Calculate the location of the centroid and show this location** on a diagram.

(b) **Calculate the second moment of area** (I_x) of the section about the horizontal centroidal axis.

(c) **Calculate the hogging bending moment** about the horizontal axis that will cause the concrete to crack, assuming that it cracks in tension at a stress of 4 MPa.

(d) **Draw the stress and strain distributions** (values required) in the cross-section due to the bending moment calculated in (c) above.

(e) Without performing any calculations, state whether the absolute value of the sagging bending moment to cause cracking is less than, equal to, or greater than the absolute value of the hogging moment calculated in (c) above. **Briefly justify the answer.**

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3) 10 marks

In Example 9.2.3 of the lecture notes, it is shown that the deflection of a simply supported beam with a UDL is given by $v = \frac{w}{24EI}\left(z^4 - 2Lz^3 + L^3 z\right)$.

Consider a 530UB82.0 steel beam, with a span of 9 m, under its self-weight only. The relevant page of the section properties handbook is given on the back page on this examination sheet – students are free to use the values given in that table without the need to recalculate them. The density of steel is 7850 kg/m^3 and the elastic modulus is E = 200000 MPa. Linear elastic properties should be assumed. The beam is oriented so that its web is in the vertical plane. Gravity may be assumed as 10 m/s^2.

(a) **What is the self-weight of the beam per unit length?**

(b) **Determine the deflection of the beam at a distance of *L*/3 along the beam**, firstly as a general algebraic expression in terms of *w*, *I*, *L*, and *E*, and secondly as a numerical value by substituting the appropriate values for those quantities.

(c) The beam is replaced by one with the same dimensions, but made from a new super lightweight alloy, which is half the density of steel, has twice the yield stress of steel, and has the same elastic modulus. **How will the deflection due to its self weight of the new alloy beam compare to the deflection previously exhibited by the original steel beam? Explain the answer.**

Part B – Applied Questions

4) 20 Marks

Consider a beam made from 610UB101 section which is oriented so that the web is in the vertical plane. A given cross section is subjected to a negative bending moment of 730 kNm about the horizontal axis, and a vertical shear force (downwards) of 600 kN. The material has linear elastic material properties, and students may use any geometric properties that are listed in the table at the end of the exam paper without calculations.

a) **Draw the stress distribution in the section due to the bending moment** (values not required). **Draw the stress distribution in the section due to the shear force** (values not required). **Explain the difference between the types of stresses** resulting from the bending moment and the shear force.

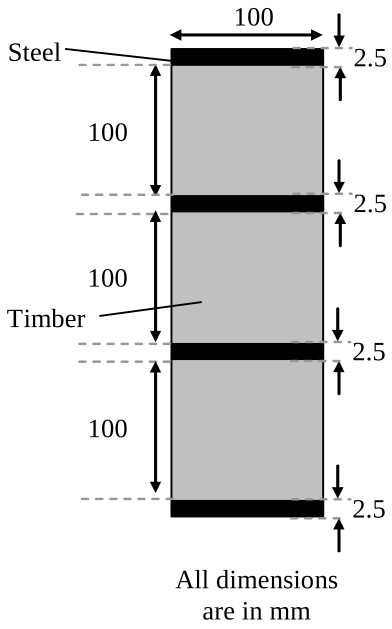
b) Consider a point at the top of the web, just below the flange web junction. **Determine all the stresses at that point, and draw a diagram to represent the stresses. Draw the Mohr's circle that represents the stresses. What are the principal stresses**?

c) Assuming the section is made from a material with yield stress f_y = 300 MPa, **determine whether the material has yielded according to both Tresca's and von Mises' yield criteria.**

d) Without performing any calculations, **comment on how the answers to b) and c) above might change** if the point under consideration was in the top flange just above the flange web junction.

5) 20 Marks

A builder tries to increase the strength of the steel/timber beam considered in Example 6.8.3 of the text book by rearranging the distribution of material. The builder uses 4 × 2.5 mm thick steel plates on each side of the timber which is cut into three equal 100 mm deep sections. It can be assumed that E_{timber} = 12500 MPa and E_{steel} = 200000 MPa. The maximum stress the timber can withstand is 4 MPa, while the maximum stress permitted in the steel is 60 MPa.



a) What is the maximum sagging bending moment the beam can resist? Does this answer differ for hogging or sagging bending moments? Explain why or why not.

b) **Draw the strain and stress distribution** due to the bending moment determined in a) above (values required).

c) This new steel beam uses the same amount of timber and steel as the beam in the original example in the text book. **Is this new beam more effective in resisting bending than the original?** It may be appropriate to compare some of the relevant properties of the two sections. **Explain** why the changes have or have not been beneficial.

6) 20 Marks

An innovation in structural engineering is the Concrete Filled Double Skinned Tube (CFDST). A CFDST consists of a hollow section placed inside another larger hollow section, and the space in between filled with concrete. The inside of the inner tube is not filled. Once filled, the CFDST acts compositely.

The CFDST has the following properties:

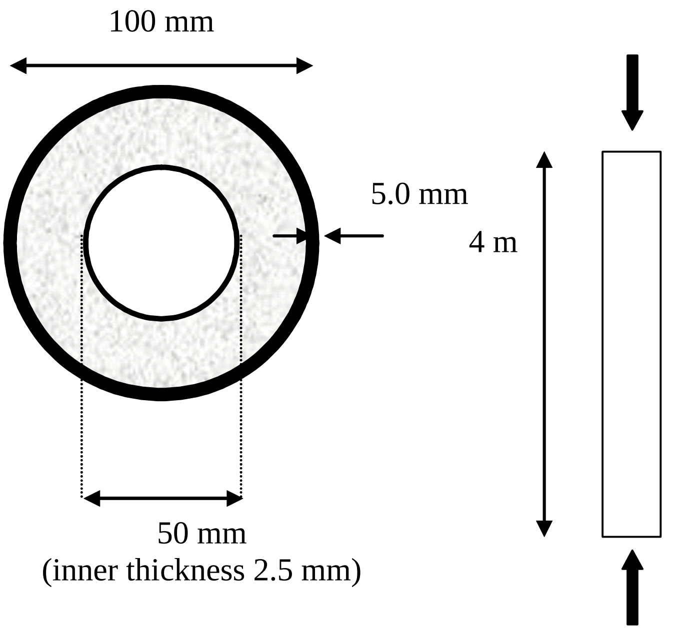
- Aluminium outer tube with dimensions 100 mm × 5 mm, and $E_{\text{aluminium}}$ = 70000 MPa.
- Steel inner tube with dimensions 50 mm × 2.5 mm, and E_{steel} = 200000 MPa.
- Concrete infill, with E_{concrete} = 30000 MPa.
- All materials behave linear elastically.
- The member is used as a column, 4 m in length, and is subjected to an axial compression.
- The design engineer wants to ensure that the stress in the concrete does not exceed $f_{\text{concrete, max}}$ = 25 MPa.

a) When the concrete reaches its maximum permissible stress of 25 MPa, give values for the following:

- Concrete strain and stress
- Aluminium strain and stress
- Steel strain and stress
- Total axial shortening
- Total axial force

b) Steel has a coefficient of thermal expansion of 12×10^{-6} (°C)$^{-1}$, concrete has a value of 12×10^{-6} (°C)$^{-1}$, and aluminium a value of 24×10^{-6} (°C)$^{-1}$.

- Ignoring any directly applied external axial load, what would happen to the steel, aluminium and concrete portions of this column if it experienced a temperature increase? How could the design of the column be changed to reduce any possible issues?



Typical column cross-section

Part C – Advanced Questions

The questions in this section are designed for students who seek to obtain a mark of Distinction or higher. They are not necessarily mathematically difficult, but require more thinking, visualisation, and application than the other questions. The majority of marks will be awarded to completed questions, and only minor marks will be awarded to partially answered questions – ie it is better to complete one question fully, than half complete 2 questions. The marks awarded in this section are bonus marks.

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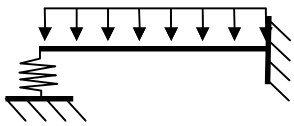
8) 10 Marks

Consider a cantilever of length L, with elastic modulus E, and second moment of area I. The tip of the cantilever rests on an elastic spring (such that the spring force is related to the spring deformation through the spring constant k). The cantilever is loaded by a UDL of magnitude w.

a) Draw the FBD of the cantilever.
b) Use compatibility to determine the reaction force between the spring and the cantilever in terms of k, w, L, E and I.
c) Determine the deflection at the cantilever tip, and draw the BMD/SFD.

Hints: Example 9.2.7.



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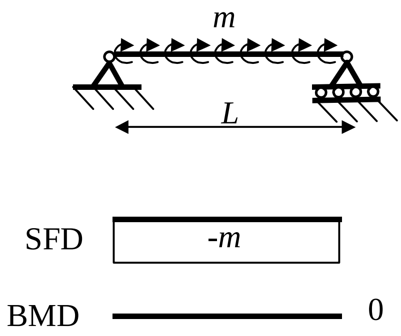
9) 10 Marks

Consider the beam shown. The shear force diagram and bending moment diagram are also shown.

m is a uniformly distributed moment (eg 10 kNm/m)



a) Draw the free body diagram of the beam and determine the reactions.

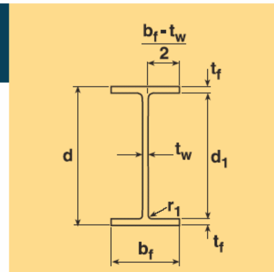
b) By making an arbitrary cut at any cross-section in the beam, and releasing the internal actions, **verify that the SFD and BMD given are correct.**

c) The moment and shear force do not satisfy the theory developed in Section 3.5 of the lecture notes: that $V = dM/dz$. **How can this be accounted for? If possible, derive an alternate differential equation that will explain this behaviour.**

[*Hint*: consider any assumptions made in Section 3.5 – are they applicable in this case?].

This is the end of the examination paper, but there is an additional data sheet on page 8.

Universal Beams



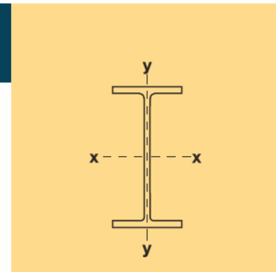


Table 15 Universal Beams — Dimensions and Properties

Designation	Depth of Section d	Flange Width b_f	Flange Thickness t_f	Web Thickness t_w	Root Radius r_1	Depth Between Flanges d_1	$\frac{d_1}{t_w}$	$\frac{(b_f-t_w)}{2t_f}$	Gross Area of Cross-Section A_g	About x-axis I_x	About x-axis Z_x	About x-axis S_x	About x-axis r_x	About y-axis I_y	About y-axis Z_y	About y-axis S_y	About y-axis r_y	Torsion Constant J	Warping Constant I_w	Designation
kg/m	mm	mm	mm	mm	mm	mm			mm^2	10^6mm^4	10^3mm^3	10^3mm^3	mm	10^6mm^4	10^3mm^3	10^3mm^3	mm	10^3mm^4	10^9mm^6	
610 UB 125	612	229	19.6	11.9	14.0	572	48.1	5.54	16000	986	3230	3680	249	39.3	343	536	49.6	1560	3450	610 UB 125
113	607	228	17.3	11.2	14.0	572	51.1	6.27	14500	875	2880	3290	246	34.3	300	469	48.7	1140	2980	113
101	602	228	14.8	10.6	14.0	572	54.0	7.34	13000	761	2530	2900	242	29.3	257	402	47.5	790	2530	101
530 UB 92.4	533	209	15.6	10.2	14.0	502	49.2	6.37	11800	554	2080	2370	217	23.8	228	355	44.9	775	1590	530 UB 92.4
82.0	528	209	13.2	9.6	14.0	502	52.3	7.55	10500	477	1810	2070	213	20.1	193	301	43.8	526	1330	82.0
460 UB 82.1	460	191	16.0	9.9	11.4	428	43.3	5.66	10500	372	1610	1840	188	18.6	195	303	42.2	701	919	460 UB 82.1
74.6	457	190	14.5	9.1	11.4	428	47.1	6.24	9520	335	1460	1660	188	16.6	175	271	41.8	530	815	74.6
67.1	454	190	12.7	8.5	11.4	428	50.4	7.15	8580	296	1300	1480	186	14.5	153	238	41.2	378	708	67.1
410 UB 59.7	406	178	12.8	7.8	11.4	381	48.8	6.65	7640	216	1060	1200	168	12.1	135	209	39.7	337	467	410 UB 59.7
53.7	403	178	10.9	7.6	11.4	381	50.1	7.82	6890	188	933	1060	165	10.3	115	179	38.6	234	394	53.7
360 UB 56.7	359	172	13.0	8.0	11.4	333	41.6	6.31	7240	161	899	1010	149	11.0	128	198	39.0	338	330	360 UB 56.7
50.7	356	171	11.5	7.3	11.4	333	45.6	7.12	6470	142	798	897	148	9.60	112	173	38.5	241	284	50.7
44.7	352	171	9.7	6.9	11.4	333	48.2	8.46	5720	121	689	777	146	8.10	94.7	146	37.6	161	237	44.7
310 UB 46.2	307	166	11.8	6.7	11.4	284	42.3	6.75	5930	100	654	729	130	9.01	109	166	39.0	233	197	310 UB 46.2
40.4	304	165	10.2	6.1	11.4	284	46.5	7.79	5210	86.4	569	633	129	7.65	92.7	142	38.3	157	165	40.4
32.0	298	149	8.0	5.5	13.0	282	51.3	8.97	4080	63.2	424	475	124	4.42	59.3	91.8	32.9	86.5	92.9	32.0
250 UB 37.3	256	146	10.9	6.4	8.9	234	36.6	6.40	4750	55.7	435	486	108	5.66	77.5	119	34.5	158	85.2	250 UB 37.3
31.4	252	146	8.6	6.1	8.9	234	38.4	8.13	4010	44.5	354	397	105	4.47	61.2	94.2	33.4	89.3	65.9	31.4
25.7	248	124	8.0	5.0	12.0	232	46.4	7.44	3270	35.4	285	319	104	2.55	41.1	63.6	27.9	67.4	36.7	25.7
200 UB 29.8	207	134	9.6	6.3	8.9	188	29.8	6.65	3820	29.1	281	316	87.3	3.86	57.5	88.4	31.8	105	37.6	200 UB 29.8
25.4	203	133	7.8	5.8	8.9	188	32.3	8.15	3230	23.6	232	260	85.4	3.06	46.1	70.9	30.8	62.7	29.2	25.4
22.3	202	133	7.0	5.0	8.9	188	37.5	9.14	2870	21.0	208	231	85.5	2.75	41.3	63.4	31.0	45.0	26.0	22.3
18.2	198	99	7.0	4.5	11.0	184	40.9	6.75	2320	15.8	160	180	82.6	1.14	23.0	35.7	22.1	38.6	10.4	18.2
180 UB 22.2	179	90	10.0	6.0	8.9	159	26.5	4.20	2820	15.3	171	195	73.6	1.22	27.1	42.3	20.8	81.6	8.71	180 UB 22.2
18.1	175	90	8.0	5.0	8.9	159	31.8	5.31	2300	12.1	139	157	72.6	0.975	21.7	33.7	20.6	44.8	6.80	18.1
16.1	173	90	7.0	4.5	8.9	159	35.3	6.11	2040	10.6	123	138	72.0	0.853	19.0	29.4	20.4	31.5	5.88	16.1
150 UB 18.0	155	75	9.5	6.0	8.0	136	22.7	3.63	2300	9.05	117	135	62.8	0.672	17.9	28.2	17.1	60.5	3.56	150 UB 18.0
14.0	150	75	7.0	5.0	8.0	136	27.2	5.00	1780	6.66	88.8	102	61.1	0.495	13.2	20.8	16.6	28.1	2.53	14.0