The University of Sydney

School of Civil Engineering

CIVL2201 Structural Mechanics

Semester 1, 2006

Time Allowed: 3 hours

Instructions to Candidates

(a) This paper contains 3 parts - A, B, & C; and is marked out of 90.

- **Part A has 3 questions (Q 1 – Q 3, 30 marks).**
- **Part B has 3 questions (Q 4 – Q 6, 60 marks).**
- **Part C has 2 questions (Q 7 - Q 8, 20 marks - which are bonus marks).**

(b) All questions may be attempted.

(c) Suitable working, diagrams and explanations are required for each question.

(d) Marks may be deducted for work that is not satisfactorily set out.

(e) Units are important.

(f) Programmable and non-programmable calculators may be used.

(g) Read the questions carefully before answering.

(h) **Annotated copies of the "Structural Mechanics" Text Book or Lecture Notes may be taken into the exam, but no other written material is permitted.** Other sheets of paper may not be inserted into the set of lecture notes.

(i) Each question has an "explain" type component. It is expected that students should answer these parts with a short written explanation (one or two paragraphs) plus a diagram. Performing calculations is not necessarily expected, but using equations might also be useful.

Part A – Fundamental Questions

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1) 10 Marks

Consider a simply supported beam with an overhang (refer to Example 3.3.4 of the text book). The length of the overhang is 2 m, while the internal span between the supports is 6 m. There is a uniformly distributed load (UDL) of 3 kN/m, acting vertically downwards along the entire length of the beam. The effects of self-weight are negligible compared to the effects of the applied loading. The beam is made from steel which has an elastic modulus of 200000 MPa.

(a) **Draw the free body diagram** of the beam clearly showing the support reactions. **Calculate** the values of the support reactions.

(b) **Draw the bending moment diagram (BMD) and shear force diagram (SFD). Clearly indicate the values** of bending moment and shear force at the key points along the beam (the supports, midpoint of the internal span, and the location of the maximum bending moment).

(c) The material of the beam is changed to one made from a new metallic alloy which has a Elastic modulus of 200 GPa. The dimensions, loading and support conditions of the beam are unchanged. **How will the BMD, SFD and deflections of the beam change?** Calculations are not required. **Briefly justify the answer.**

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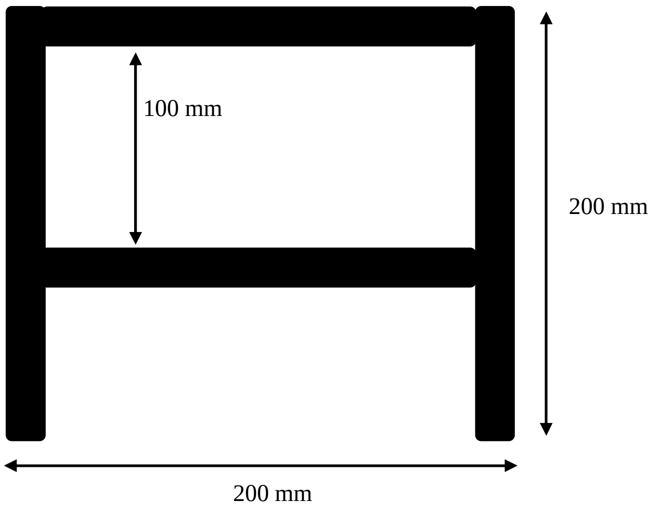
2) 10 Marks

Consider the following modified box section, the dimensions of which are shown in the diagram. The section is symmetric about the vertical axis. The cross section is made from aluminium which should be assumed to be linear elastic with an elastic modulus of 70000 MPa, and a yield stress of 280 MPa.



All elements are 10 mm thick

(a) **Calculate the location of the centroid and show this location** on a diagram.

(b) **Calculate the second moment of area** (I_x) of the section about the horizontal centroidal axis.

(c) **Calculate the bending moment** about the horizontal axis that will cause first yield of the section.

(d) **Draw the stress and strain distributions** (values required) in the cross-section due to the bending moment calculated in (c) above.

(e) The thicknesses of the horizontal elements are slightly decreased by the same amount, while the thicknesses of the vertical elements remain unchanged. **How will the location of the centroid change in both the vertical and horizontal directions?** Calculations are not required. **Briefly justify the answer.**

3) 10 marks

In Example 9.2.3 of the lecture notes, it is shown that the deflection of a simply supported beam with a UDL is given by $v = \frac{w}{24EI}\left(z^4 - 2Lz^3 + L^3 z\right)$.

Consider a 180UB22.2 simply supported steel beam, with unknown length L. The relevant page of the section properties handbook is given on the back page on this examination sheet – students are free to use the values given in that table without the need to recalculate them. The density of steel is 7850 kg/m^3 and the elastic modulus is E = 200000 MPa. The beam is oriented so that its web is in the vertical plane. The beam experiences a uniformly distributed load, w, acting vertically downwards along the entire length of the beam. Gravity may be assumed as 10 m/s^2.

(a) **Draw a diagram** that shows the beam layout, the loading, and the support conditions**;** and also **provide a sketch** that shows the beam cross section at midspan.

(b) For selfweight only, **determine the length of the beam** such that the maximum deflection is equal to L/2000.

(c) Assume the beam is no longer made of steel, but of a new material with unknown density but the same elastic modulus, and that the beam is 4 m long. **What new material density** would cause a deflection due to selfweight only of 2 mm at the point 1 m from the left hand support?

(d) The beam is reoriented so that the web of the beam is now in the horizontal plane. **How would the answers to parts (b) and (c) change?** Calculations are not required. **Briefly justify the answer.**

Part B – Applied Questions

4) 20 Marks

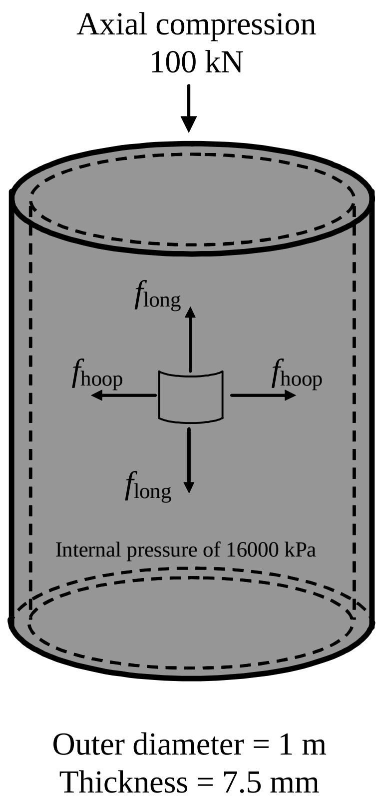
Consider a closed cylindrical pressure vessel. The vessel has an outer diameter of 1 m and a thickness of 7.5 mm, and is subjected to an internal pressure of 16000 kPa. In additional the vessel is subjected to a vertical compressive load of 100 kN.

(a) **Calculate the hoop stress, longitudinal stress and shear stress** resulting from the loads mentioned above and experienced by a typical element within the vessel as shown in the attached diagram.

(b) **Calculate the principal stresses and draw the Mohr's circle** that represents the stresses calculated in a) above.

(c) Assuming the pressure vessel is made from a material with yield stress f_y = 1100 MPa, **determine whether the material has yielded** according to both Tresca's and von Mises' yield criteria.

(d) The material is replaced by another linear elastic material with the same yield stress, but an elastic modulus that is twice the original magnitude. **How would the above answers change?** Calculations are not expected but a brief explanation is required.



5) 20 Marks

A concrete beam, length 5 m, with square cross-section 300 mm × 300 mm is symmetrically reinforced with 6 steel reinforcing bars, each of diameter 16 mm. The centre of each corner reinforcing bar is located 60 mm from closest edges of the concrete beam. The beam is simply supported and experiences a uniformly distributed load along its entire length. It can be assumed that $E_{concrete} = 40$ GPa and $E_{steel} = 200000$ MPa and that both materials exhibit linear elastic behaviour. The concrete cracks if the tensile stress in the concrete exceeds 4 N/mm^2. Consider the cross-section of the beam that is experiencing the greatest positive bending moment just as the concrete is about to crack.

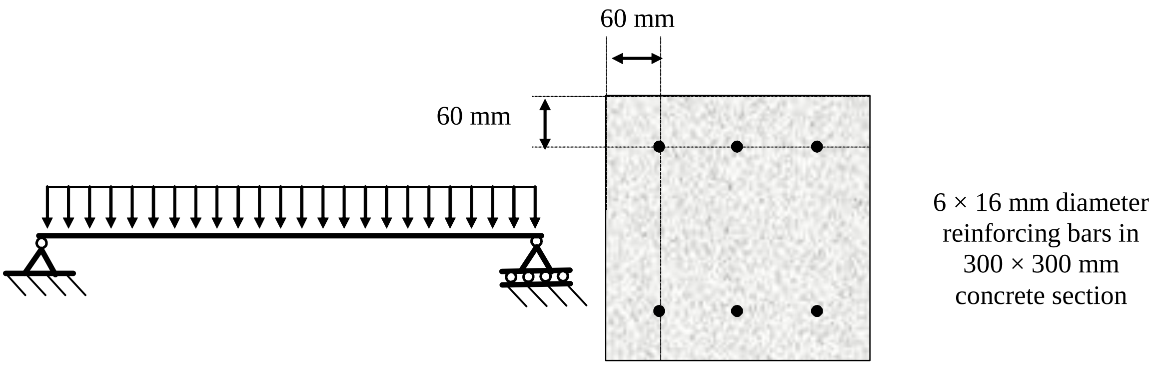
a) Using a diagram, **identify the location** along the length of the beam at which a cross section would experience the greatest values of bending stress. On another diagram **show which point(s) on this section will crack first**. **Is the beam hogging or sagging** at this point?

b) **What is the strain in the concrete just before it cracks? Draw the strain distribution** across this critical section (values required) just before cracking. For the steel reinforcement it is sufficient to give the strain at the centre of the reinforcing bar.

c) **Draw the stress distribution** across this critical section (values required). For the steel reinforcement it is sufficient to give the stress at the centre of the reinforcing bar.

d) **What is the bending moment that will cause the concrete to crack? What is the value of uniformly distributed load that will produce this bending moment?**

e) The upper and lower rows of reinforcing bars are moved slightly closer towards the centre of the cross-section to give increased concrete cover, which improves fire resistance. **How would the cracking moment (part d) change?** Calculations are not expected but a brief explanation is required.



6) 20 Marks

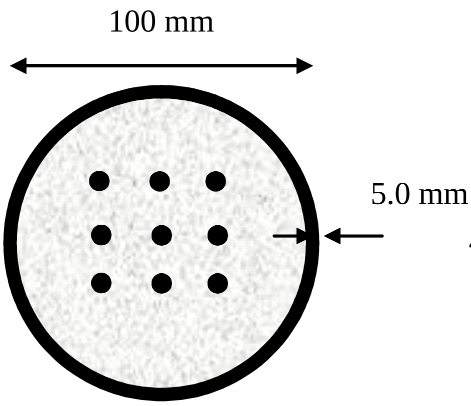
Consider a 4 m column, the cross section of which is a 100 × 5 CHS made from steel. The steel tube is filled with concrete and 9 symmetrically placed 10 mm diameter steel reinforcing bars. Once filled, the steel tube, steel reinforcing and the concrete act compositely. It can be assumed that $E_{concrete}$ = 40000 MPa and E_{steel} = 200000 MPa and that both materials behave linear elastically.

The design engineer wants to ensure that the axial compression in an individual reinforcing rod does not exceed $P_{reinforcing,\ max}$ = 10 kN.

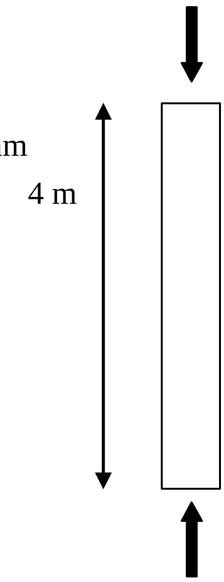
a) **At what value of total axial compression force on the entire column** does an individual steel reinforcing bar reach its maximum permissible load of 10 kN?

b) **For the value of load calculated in part (a) above, give the following**:

- Steel strain
- Steel stress
- Concrete strain
- Concrete stress
- Total axial shortening



Typical column cross-section



c) The steel reinforcing bars are removed and the gaps filled with concrete. Assuming that the same load as calculated in (a) above is applied, would the following increase or decrease (compared to the original case)? Explain your answers.

- Steel strain
- Steel stress
- Concrete strain
- Concrete stress
- % of the overall load carried by the concrete
- % of the overall load carried by the steel
- Total axial shortening

Part C – Advanced Questions

The questions in this section are designed for students who seek to obtain a mark of Distinction or higher. They are not necessarily mathematically difficult, but require more thinking, visualisation, and application than the other questions. The majority of marks will be awarded to completed questions, and only minor marks will be awarded to partially answered questions – ie it is better to complete one question fully, than half complete 2 questions.

The marks awarded in this section are bonus marks.

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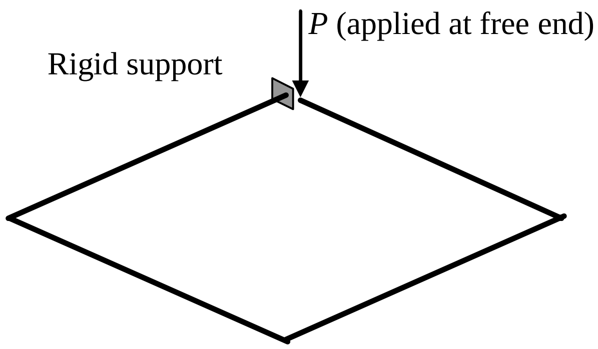
8) 10 Marks

The (almost) square shaped structure is made up of four components, each of length *L*. The structure lies in a horizontal plane and each segment is perpendicular to the adjacent segment. The support is completely fixed, preventing deflection in every direction, and rotation about any axis. The structure is not a closed square – the end of the 4th element is free, and there is a small distance (which should be ignored in any calculations) between the free end and the support. There is a vertical load of magnitude *P* acting at the free end of the structure. **Draw the bending moment, twisting moment, shear force and axial force diagrams for the structure.**



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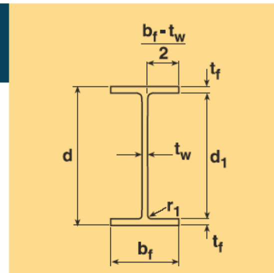
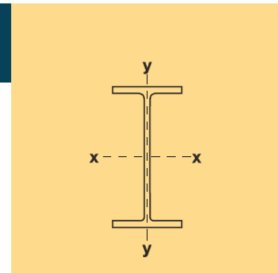
9) 10 Marks

Consider a circular hollow section with outer diameter 200 mm and thickness 5 mm. It is made from steel with linear elastic – perfectly plastic material properties and a yield stress of 400 MPa. **Calculate the shape factor and plastic moment of the cross section.**

This is the end of the examination paper, but there is an additional data sheet on page 8.





Universal Beams

Table 15 Universal Beams — Dimensions and Properties

Designation	Depth of Section d	Flange Width b_f	Flange Thickness t_f	Web Thickness t_w	Root Radius r_1	Depth Between Flanges d_1	$\frac{d_1}{t_w}$	$\frac{(b_f-t_w)}{2t_f}$	Gross Area of Cross-Section A_g	About x-axis I_x	Z_x	S_x	r_x	About y-axis I_y	Z_y	S_y	r_y	Torsion Constant J	Warping Constant I_w	Designation
kg/m	mm	mm	mm	mm	mm	mm			mm^2	10^6mm^4	10^3mm^3	10^3mm^3	mm	10^6mm^4	10^3mm^3	10^3mm^3	mm	10^3mm^4	10^9mm^6	
610 UB 125	612	229	19.6	11.9	14.0	572	48.1	5.54	16000	986	3230	3680	249	39.3	343	536	49.6	1560	3450	610 UB 125
113	607	228	17.3	11.2	14.0	572	51.1	6.27	14500	875	2880	3290	246	34.3	300	469	48.7	1140	2980	113
101	602	228	14.8	10.6	14.0	572	54.0	7.34	13000	761	2530	2900	242	29.3	257	402	47.5	790	2530	101
530 UB 92.4	533	209	15.6	10.2	14.0	502	49.2	6.37	11800	554	2080	2370	217	23.8	228	355	44.9	775	1590	530 UB 92.4
82.0	528	209	13.2	9.6	14.0	502	52.3	7.55	10500	477	1810	2070	213	20.1	193	301	43.8	526	1330	82.0
460 UB 82.1	460	191	16.0	9.9	11.4	428	43.3	5.66	10500	372	1610	1840	188	18.6	195	303	42.2	701	919	460 UB 82.1
74.6	457	190	14.5	9.1	11.4	428	47.1	6.24	9520	335	1460	1660	188	16.6	175	271	41.8	530	815	74.6
67.1	454	190	12.7	8.5	11.4	428	50.4	7.15	8580	296	1300	1480	186	14.5	153	238	41.2	378	708	67.1
410 UB 59.7	406	178	12.8	7.8	11.4	381	48.8	6.65	7640	216	1060	1200	168	12.1	135	209	39.7	337	467	410 UB 59.7
53.7	403	178	10.9	7.6	11.4	381	50.1	7.82	6890	188	933	1060	165	10.3	115	179	38.6	234	394	53.7
360 UB 56.7	359	172	13.0	8.0	11.4	333	41.6	6.31	7240	161	899	1010	149	11.0	128	198	39.0	338	330	360 UB 56.7
50.7	356	171	11.5	7.3	11.4	333	45.6	7.12	6470	142	798	897	148	9.60	112	173	38.5	241	284	50.7
44.7	352	171	9.7	6.9	11.4	333	48.2	8.46	5720	121	689	777	146	8.10	94.7	146	37.6	161	237	44.7
310 UB 46.2	307	166	11.8	6.7	11.4	284	42.3	6.75	5930	100	654	729	130	9.01	109	166	39.0	233	197	310 UB 46.2
40.4	304	165	10.2	6.1	11.4	284	46.5	7.79	5210	86.4	569	633	129	7.65	92.7	142	38.3	157	165	40.4
32.0	298	149	8.0	5.5	13.0	282	51.3	8.97	4080	63.2	424	475	124	4.42	59.3	91.8	32.9	86.5	92.9	32.0
250 UB 37.3	256	146	10.9	6.4	8.9	234	36.6	6.40	4750	55.7	435	486	108	5.66	77.5	119	34.5	158	85.2	250 UB 37.3
31.4	252	146	8.6	6.1	8.9	234	38.4	8.13	4010	44.5	354	397	105	4.47	61.2	94.2	33.4	89.3	65.9	31.4
25.7	248	124	8.0	5.0	12.0	232	46.4	7.44	3270	35.4	285	319	104	2.55	41.1	63.6	27.9	67.4	36.7	25.7
200 UB 29.8	207	134	9.6	6.3	8.9	188	29.8	6.65	3820	29.1	281	316	87.3	3.86	57.5	88.4	31.8	105	37.6	200 UB 29.8
25.4	203	133	7.8	5.8	8.9	188	32.3	8.15	3230	23.6	232	260	85.4	3.06	46.1	70.9	30.8	62.7	29.2	25.4
22.3	202	133	7.0	5.0	8.9	188	37.5	9.14	2870	21.0	208	231	85.5	2.75	41.3	63.4	31.0	45.0	26.0	22.3
18.2	198	99	7.0	4.5	11.0	184	40.9	6.75	2320	15.8	160	180	82.6	1.14	23.0	35.7	22.1	38.6	10.4	18.2
180 UB 22.2	179	90	10.0	6.0	8.9	159	26.5	4.20	2820	15.3	171	195	73.6	1.22	27.1	42.3	20.8	81.6	8.71	180 UB 22.2
18.1	175	90	8.0	5.0	8.9	159	31.8	5.31	2300	12.1	139	157	72.6	0.975	21.7	33.7	20.6	44.8	6.80	18.1
16.1	173	90	7.0	4.5	8.9	159	35.3	6.11	2040	10.6	123	138	72.0	0.853	19.0	29.4	20.4	31.5	5.88	16.1
150 UB 18.0	155	75	9.5	6.0	8.0	136	22.7	3.63	2300	9.05	117	135	62.8	0.672	17.9	28.2	17.1	60.5	3.56	150 UB 18.0
14.0	150	75	7.0	5.0	8.0	136	27.2	5.00	1780	6.66	88.8	102	61.1	0.495	13.2	20.8	16.6	28.1	2.53	14.0